Mead

COMPOSITION

LOGBOOK # 61

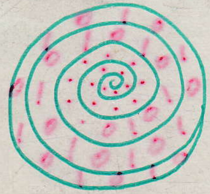
JDi: 2451585.7 JDf: 2451625.4

START: 11 FEB 2000 END: 21 MAR

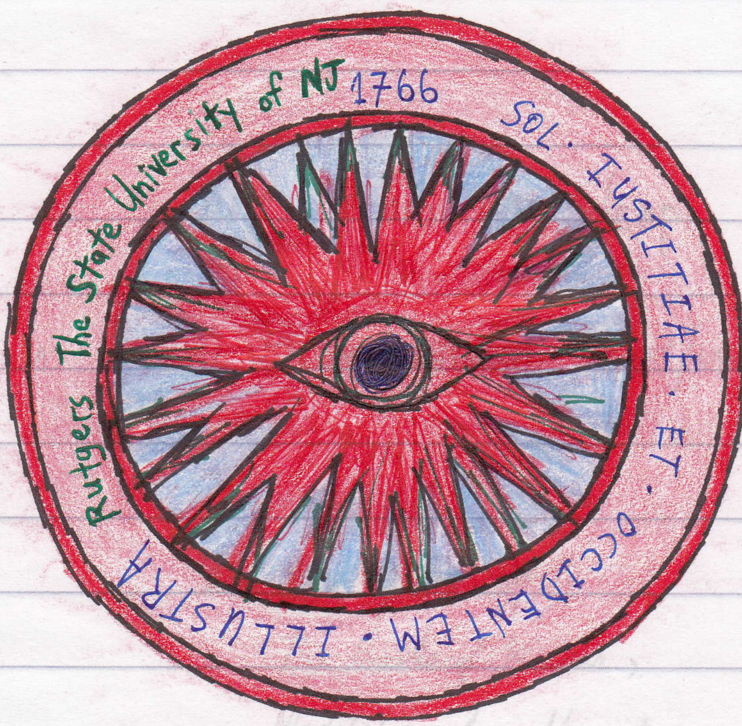
152 sheets • 304 pages

$9\frac{3}{4}$ x $7\frac{1}{2}$ in/24.7 x 19.0 cm

wide ruled • 09710



OUT FROM THE DEEP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2000 C.E.

out from the deep

A MICHAEL WILLIAM HENTRICH DIARY/LOGBOOK

In the spirit of:
The Books of Wonder,
Meditations of a Hermit,
Notes From The Abyss,
Scribblings, Brainware,
L3E ...

Logbook 61

CONTENTS

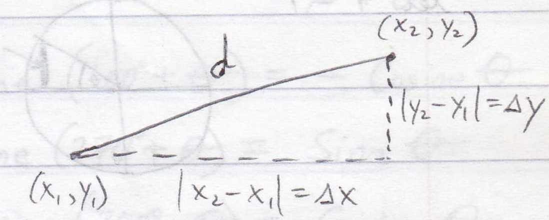
MATHEMATICS REVIEW 2000

Geometry

COGITATION #063 If one side of a triangle is produced, then the exterior angle is equal to the sum of the two interior opposite angles.

$$\theta = \alpha + \beta$$

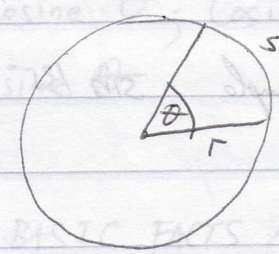
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

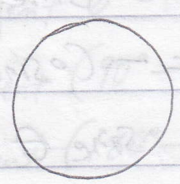


radian measure:

$$s = r\theta$$

$$\theta = \frac{s}{r}$$



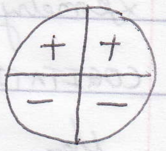


sphere surface area $= 4\pi r^2$

volume $= \dfrac{4\pi r^3}{3}$

Trigonometry

COGITATION #064 BASIC FACTS ABOUT THE SINE

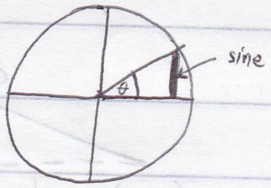


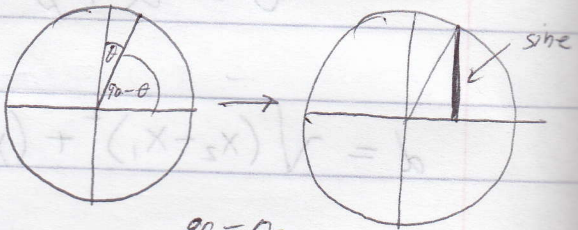
Sine $0° = \emptyset$; Sine $180° = \emptyset$; Sine $90° = 1$; Sine $270° = -1$

multiples of 90 degrees plus or minus a given angle :

Sine $(90° - \theta)$ = Cosine θ ; Sine $(90° + \theta)$ = Cosine θ

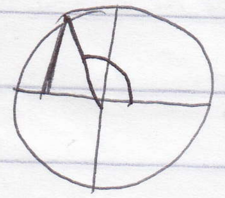
let $r = 1$





90 − θ

90 + θ



example sin 135° = cos 45°

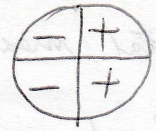
Sine $(180° - \theta)$ = Sine θ ; Sine $(180° + \theta)$ = − Sine θ

Sine $(270° - \theta)$ = − Cosine θ ; Sine $(270° + \theta)$ = − Cosine θ

Sine $(360° - \theta)$ = − Sine θ ; Sine $(360° + \theta)$ = Sine θ

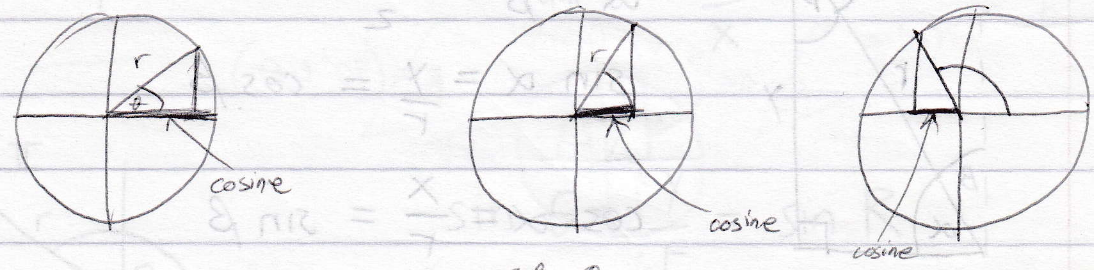
Sine $(-\theta)$ = − Sine θ

COGITATION # 065 BASIC FACTS ABOUT THE COSINE



Cosine $0° = 1$; Cosine $180° = -1$; Cosine $90° = \emptyset$; Cosine $270° = \emptyset$

Cosine $(90° - \theta)$ = Sine θ; Cosine $(90° + \theta)$ = − Sine θ



$90° - \theta$ $\quad$ $90° + \theta$

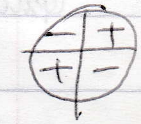
Cosine $(180° - \theta)$ = − Cosine θ; Cosine $(180° + \theta)$ = − Cosine θ

Cosine $(270° - \theta)$ = − Sine θ; Cosine $(270° + \theta)$ = Sine θ

Cosine $(360° - \theta)$ = Cosine θ; Cosine $(360° + \theta)$ = Cosine θ

Cosine $(-\theta)$ = Cosine θ

COGITATION # 066 BASIC FACTS ABOUT THE TANGENT



Tangent $0°$ = Tangent $180° = \emptyset$; Tangent $90°$ = Tangent $270° = \infty$

Tangent $(90° - \theta)$ = Cotangent θ; Tangent $(90° + \theta)$ = − Cotangent θ

Tangent $(180° - \theta)$ = − Tangent θ; Tangent $(180° + \theta)$ = Tangent θ

Tangent $(270° - \theta)$ = Cotangent θ; Tangent $(270° + \theta)$ = − Cotangent

Tangent $(360° - \theta)$ = − Tangent θ; Tangent $(360° + \theta)$ = Tangent θ

Tangent $(-\theta)$ = − Tangent θ

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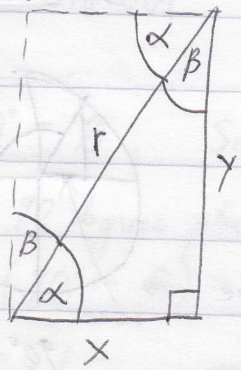
COGITATION # ~~066~~ 067

complementary angles → total measure is 90°

The sine of an angle equals the cosine of its complementary angle.

proof:



$$\alpha + \beta = \frac{\pi}{2}$$

$$\sin \alpha = \frac{Y}{r} = \cos \beta$$

$$\cos \alpha = \frac{X}{r} = \sin \beta$$

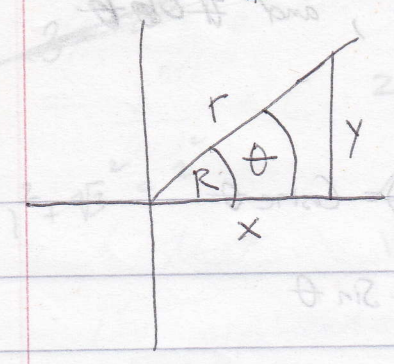
$$\tan \alpha = \frac{Y}{X} = \cot \beta$$

COGITATION # 068 Reference angles

Quadrant	Relationship	Function signs
I	$R = \theta$	all +
II	$R = 180° - \theta$	sin +, cos −, tan −
III	$R = \theta - 180°$	sin −, cos −, tan +
IV	$R = 360° - \theta$	sin −, cos +, tan −

sin + cos −	all +
sin − cos −	sin − cos +

I

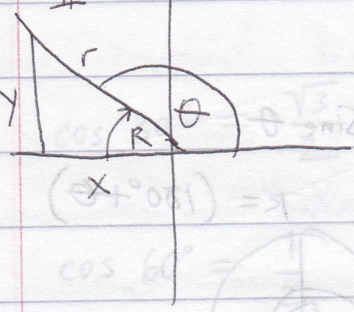


$$\sin\theta = \frac{y}{r}$$

$$\cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x}$$

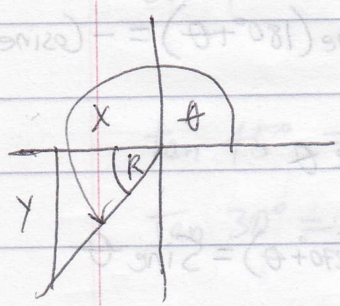
II



$$\sin\theta = \frac{y}{r} = \sin R$$

$$\cos\theta = \frac{-x}{r} = -\cos R$$

$$\tan\theta = \frac{y}{-x} = -\tan R$$

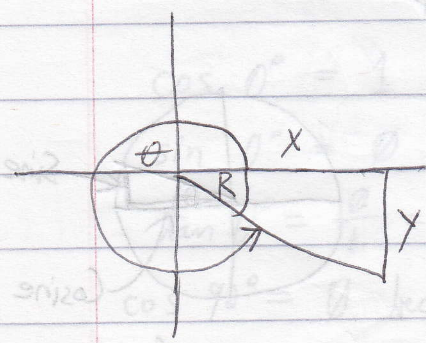


III

$$\cancel{\cos\theta}\ \sin\theta = \frac{-y}{r} = -\sin R$$

$$\cos\theta = \frac{-x}{r} = -\cos R$$

$$\tan\theta = \frac{-y}{-x} = \tan R$$



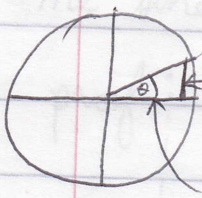
IV

$$\sin\theta = \frac{-y}{r} = -\sin R$$

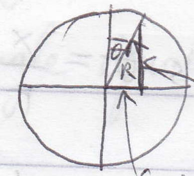
$$\cos\theta = \frac{x}{r} = \cos R$$

$$\tan\theta = \frac{-y}{x} = -\tan R$$

Back to Cogitations #064, #065, and #066



$R = 90° - \theta$

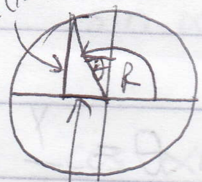


Sine $(90° - \theta) =$ Cosine θ

Cosine $(90 - \theta) =$ Sin θ

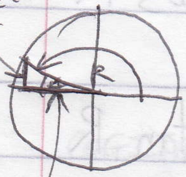
Sine $(90° + \theta) =$ Cosine θ

$R = (90° + \theta)$



Cosine $(90° + \theta) = -$ Sine θ

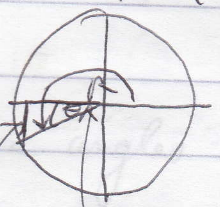
Sine $(180 - \theta) =$ Sine θ



$R = (180° - \theta)$

Cosine $(180° - \theta) = -$ Cosine θ

$R = (180° + \theta)$

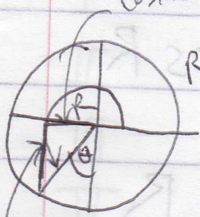


Sine $(180° + \theta) = -$ Sine θ

Cosine $(180° + \theta) = -$ Cosine θ

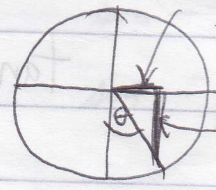
Cosine $(270° - \theta) = -$ Sine θ

$R = (270° - \theta)$



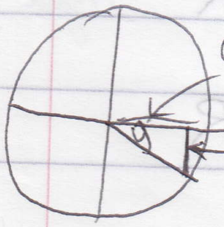
Sine $(270° - \theta) = -$ Cosine θ

Cosine $(270° + \theta) =$ Sine θ

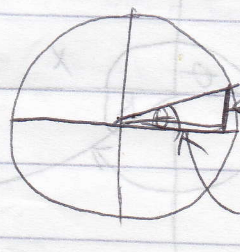


Sine $(270° + \theta) = -$ Cosine θ

Cosine $(360° - \theta) =$ Cosine θ



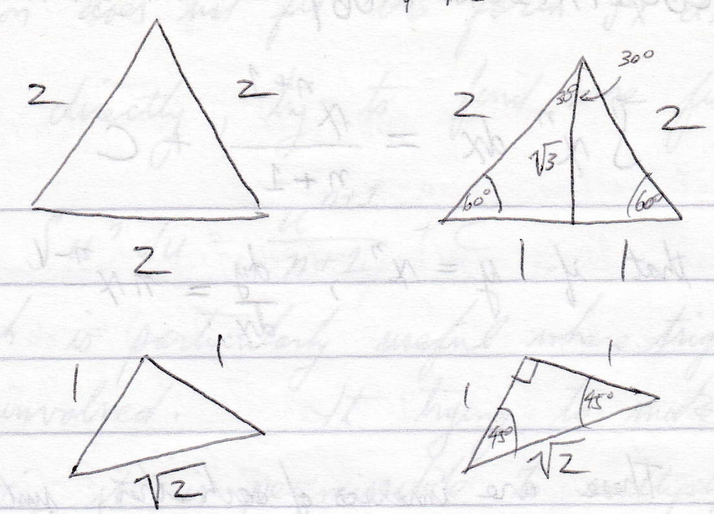
Sine $(360° - \theta) = -$ Sine θ



original

$2^2 - 1^2 = 3$*

$1^2 + \sqrt{3}^2 = 2^2$



$\cos 30° = \frac{\sqrt{3}}{2} = \sin 60° = 0.866$

$\cos 60° = \frac{1}{2} = \sin 30° = 0.5$

$\cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \sin 45° \approx 0.707$

$\tan 45° = 1 = \frac{\sin 45°}{\cos 45°}$ ~~tan θ~~

$\tan 30° = \frac{\sin 30°}{\cos 30°} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \approx 0.577$

$\tan 60° = \frac{\sin 60°}{\cos 60°} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3} = 1.73$

cos 0° = 1 because hypotenuse IS adjacent line $\longrightarrow$ angle zero

sin 0° = 0 because at 0° $\longrightarrow$, there is no opposite angle

tan 0° = $\frac{0}{1}$ = 0 ; whereas tan 90° is UNDEFINED: $\frac{1}{0}$ DNE !

cos 90° = 0 because adjacent ~~angle~~ side DNE (does not exist)

sin 90° = 1 because hypotenuse IS opposite side.

COGITATION # 069 FUNDAMENTAL INTEGRATION

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$$

note that if $y = x^n$, $\frac{dy}{dx} = n x^{n-1}$

These are inverses of each other, just as subtraction is the inverse function of addition.

Note the players, the cast, the notation:

Differentiation	Integration
$\cancel{\int x^n \, dx}$	
$\frac{d(x^n)}{dx}$	$\int x^n \, dx$

$d(x^n) \longleftrightarrow \int x^n$

$\overline{\quad dx \quad} \longleftrightarrow dx$

$n * x^{n-1} \longleftrightarrow \frac{x^{n+1}}{n+1}$

If the function does not fit the form $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ directly, try to find some function u, such that $\int u^n du = \frac{u^{n+1}}{n+2} + C$.

This approach is particularly useful where trigonometric functions are involved. It trying to make functions fit this form, it is permissible to multiply by any constant UNDER the integral sign, while, at the same time, to leave the function unchanged, we have to divide by the same constant OUTSIDE the integral sign.

Integration by parts is attempted when the above method fails.

$$\int u\,dv = uv - \int v\,du$$

$$\int \sqrt{10^{3x}}\,dx = \int 10^{\frac{3x}{2}}\,dx$$

$\int a^u du = \frac{a^u}{\ln a}$ applies. $a = 10$ $\quad u = \frac{3x}{2}$

$du = \frac{3}{2}\,dx$

$$\frac{2}{3}\frac{10^u}{\ln 10} + C = \frac{2}{3}\frac{10^{3x/2}}{\ln 10} + C$$

COGITATION # 070 The field of real numbers

	ADDITION	MULTIPLICATION
The commutative law:	$a+b=b+a$	$ab=ba$
The associative law:	$a+(b+c)=(a+b)+c$	$a(bc)=(ab)c$

The distributive law: $a(b+c)=ab+ac$

$a+\emptyset=a$; $a\cdot 1=a$

additive inverse: $a+x=\emptyset$ $\{-a\}$

multiplicative inverse: $ax=1$ $\{a^{-1}=\frac{1}{a}\}$

$a-b=a+(-b)$; $a/b=a(b^{-1})$

if a and b are in F, then $a+b$ and ab are in F.

The commutative, associative, and distributive laws hold.

F contains $\emptyset$ and 1

COGITATION # 071 Taylor's Formula With integral remainder

$P(x)=b_0x^n+b_1x^{n-1}+\dots+b_n$ $(n\geq\emptyset)$ where $b_0\neq\emptyset$

if $x=a$, then $P(x)$ is the sum of powers of $(x-a)$, the highest power being n:

$$P(x)=c_0(x-a)^n+c_1(x-a)^{n-1}+\dots+c_n$$

$$P^{(k)}(a)=\left\{\frac{d^k}{dx^k}\left[c_{n-k}(x-a)^k\right]\right\}_{x=a}=k!\,c_{n-k},$$

consequently $c_0=\dfrac{P^{(n)}(a)}{n!}$, $c_1=\dfrac{P^{(n-1)}(a)}{(n-1)!}$, ..., $c_n=P(a)$

hence, $P(x) = c_0(x-a)^n + c_1(x-a)^{n-1} + \dots + c_n$ may be rewritten as:

$$P(x) = P(a) + P'(a)(x-a) + \frac{P''(a)}{2!}(x-a)^2 + \dots + \frac{P^{(n)}(a)}{n!}(x-a)^n,$$

where we have reversed the order of the terms.

What relation does the expression

$$f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

bear to $f(x)$?

Assume f can be differentiated n times at $x=a$. Assume even more than this. Assume that f and its first $n+1$ derivatives are continuous in a closed interval containing $x=a$.

by the second fundamental theorem of calculus,

Let f be a given function continuous on the closed interval $[a,b]$. Suppose that F is any differentiable function such that $F'(x) = f(x)$ when $a \le x \le b$. Then

$$\int_a^b f(x)\,dx = F(b) - F(a),$$

we know that

$$\int_a^x f'(t)\,dt = f(x) - f(a)$$

Let us write this in the form $f(x) = f(a) + \int_a^x f'(t)\,dt$

We transform the integral by integration by parts, taking

$u = f'(t)$, $du = f''(t)dt$, $dv = dt$, $v = -(x-t)$

thus, $\int_a^x f'(t)\,dt = -f'(t)(x-t)\Big|_a^x + \int_a^x f''(t)(x-t)\,dt$,

and so, $f(x) = f(a) + f'(a)(x-a) + \int_a^x f''(t)(x-t)\,dt$.

We can now integrate by parts again.

ix $u = f''(t)$, $du = f^{(3)}(t)\,dt$

$dv = (x-t)\,dt$, $v = \dfrac{-(x-t)^2}{2!}$

The integral now becomes

$$\int_a^x f''(t)(x-t)\,dt = -f''(t)\frac{(x-t)^2}{2!}\Big|_a^x + \int_a^x f^{(3)}(t)\frac{(x-t)^2}{2!}$$

and so,

$$f(x) = f(a) + f'(a)(x-a) + f''(a)\frac{(x-a)^2}{2!} + \frac{1}{2!}\int_a^x f^{(3)}(t)(x-t)^2$$

Repeated integration by parts will lead to the formula

$$f(x) = f(a) + f'(a)(x-a) + \ldots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$+ \frac{1}{n!}\int_a^x (x-t)^n f^{(n+1)}(t)\,dt$$

The function $f(x)$ has been expressed as a polynomial of degree n in $(x-a)$, plus a remainder term.

We can state our findings in a formal theorem.

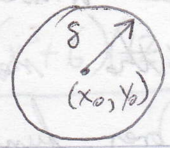
Let $f(x)$ and its first $n+1$ derivatives $(n \geq \emptyset)$ be continuous in a closed interval containing $x=a$. Let x be any point of this interval. Then

$$f(x) = f(a) + f'(a)(x-a) + \ldots + \frac{f^{(n)}(a)}{n!}(x-a)^n + R_{n+1}$$

the remainder $R_{n+1} = \frac{1}{n!}\int_a^x (x-t)^n f^{(n+1)}(t)\,dt$

COGITATION # 072 Functions of Several Variables

Point Sets



Limits

$$\lim_{(x,y) \to (x_0, y_0)} f(x,y) = A$$

Modes of representing a function:

we write $y = f(x)$ and plot the points (x, y)

if $f(x)$ is continuous on an interval, the graph will be a curve in the plane.

we write $z = f(x, y)$ and plot the points (x, y, z)

if f is continuous in a region R of the xy plane, we obtain a surface in space.

When we go to 3 independent variables, there is no satisfactory analogue of the foregoing methods of graphical representation, for we cannot draw upon any familiar geometric intuition to visualize $w = f(x, y, z)$ as ~~configuring~~ defining a configuration in space of four dimensions.

Note: It is at this point I pause. I would be wise to

wait until I get to Rutgers before proceeding with multivariable calculus. I ~~I~~ NOW want to use whatever moments of inspiration that come my way over the next couple of weeks [$L_{60} \to 2000.001..018$] to go over fundamental ideas in basic calculus and linear algebra.

COGITATION # 073 — Some rules for differentiation

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$y = (x^3+5)(6x^5+7)$$

$$\frac{dy}{dx} = (x^3+5)(30x^4) + (6x^5+7)(3x^2)$$

$$= 30x^7 + 150x^4 + 18x^7 + 21x^2 = 48x^7 + 150x^4 + 21x^2$$

Note that the same solution is arrived at when we choose to multiply before differentiating:

$$y = 6x^8 + 30x^5 + 7x^3 + 35$$

$$\frac{dy}{dx} = 48x^7 + 150x^4 + 21x^2$$

---

for quotients: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

In words: (a) Multiply the denominator v by the derivative of the numerator $\left(\frac{du}{dx}\right)$.

(b) Multiply the numerator by the derivative of the denominator (c) $\frac{(a)-(b)}{v^2}$

example: $y = \dfrac{3x^4+5}{x^3+6}$

$$\frac{dy}{dx} = \frac{(x^3+6)(12x^3) - (3x^4+5)(3x^2)}{(x^3+6)^2}$$

$$= \frac{12x^6 + 72x^3 - 9x^6 - 15x^2}{x^6 + 12x^3 + 36} = \frac{3x^6 + 72x^3 - 15x^2}{(x^3+6)^2}$$

---

if $y = x^{3b}$, $\dfrac{dy}{dx} = 3b\, x^{3b-1}$

---

if $y = \sqrt[a]{\dfrac{1}{x^c}}$, $\dfrac{dy}{dx} = -\dfrac{c}{a} \cdot x^{-\frac{c}{a}-1} = -\dfrac{c}{a} x^{-\frac{(c+a)}{a}}$

$\left[y = \left(\dfrac{1}{x^c}\right)^{1/a} = x^{-c/a} \right]$; $\dfrac{dy}{dx} = -\dfrac{c}{a} \sqrt[a]{\dfrac{1}{x^{c+a}}}$

---

if $v = \sqrt[4]{\dfrac{1}{y^7}} = y^{-7/4}$

$$\frac{dv}{dy} = -\frac{7}{4} \cdot y^{-\frac{7}{4}-1} = -\frac{7}{4} y^{-\frac{7}{4}-\frac{4}{4}} = -\frac{7}{4} y^{-11/4}$$

$$= -\frac{7}{4} \sqrt[4]{\frac{1}{y^{11}}}$$

---

if $y = \dfrac{ax^n - 2}{b} = \dfrac{a}{b} x^n - \dfrac{2}{b}$

$$\frac{dy}{dx} = \frac{a}{b} \cdot n \cdot x^{n-1} - \emptyset = \frac{na}{b} \cdot x^{n-1}$$

COGITATION # 074 SUBSTITUTION

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

example $\quad y = (x^3 + b^4)^{5/2}$

Let $(x^3 + b^4) = u$

then $y = u^{5/2}$

and $\dfrac{dy}{du} = \dfrac{5}{2} u^{\frac{5}{2} - \frac{2}{2}} = \dfrac{5}{2} u^{3/2}$

also $\dfrac{du}{dx} = \dfrac{d}{dx}(x^3 + b^4) = 3x^2$

$$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{5}{2} u^{3/2} \cdot 3x^2$$

$$= \frac{5}{2}(x^3 + b^4)^{3/2} \cdot 3x^2$$

$$= \frac{15}{2} x^2 (x^3 + b^4)^{3/2}$$

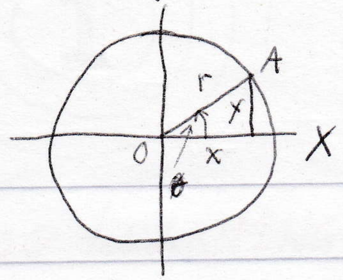
example $\quad y = \sqrt{b + x} \qquad$ let $u = (b + x)$

then $y = \sqrt{u} = u^{1/2}$

$\dfrac{dy}{du} = \dfrac{1}{2} u^{-1/2}$; also $\dfrac{du}{dx} = 1$

$$\therefore \frac{dy}{dx} = \frac{1}{2} u^{-1/2} \cdot 1 = \frac{1}{2}(b + x)^{-1/2} = \frac{1}{2\sqrt{b + x}}$$

$$x = r\cos\theta$$
$$y = r\sin\theta$$



Eliminating the parameter θ by squaring and adding the parametric equations,

$$r^2\cos^2\theta + r^2\sin^2\theta = x^2 + y^2$$

$$r^2(\cos^2\theta + \sin^2\theta) = x^2 + y^2$$

$$[\cos^2\theta + \sin^2\theta = 1]$$

$$r^2 = x^2 + y^2$$

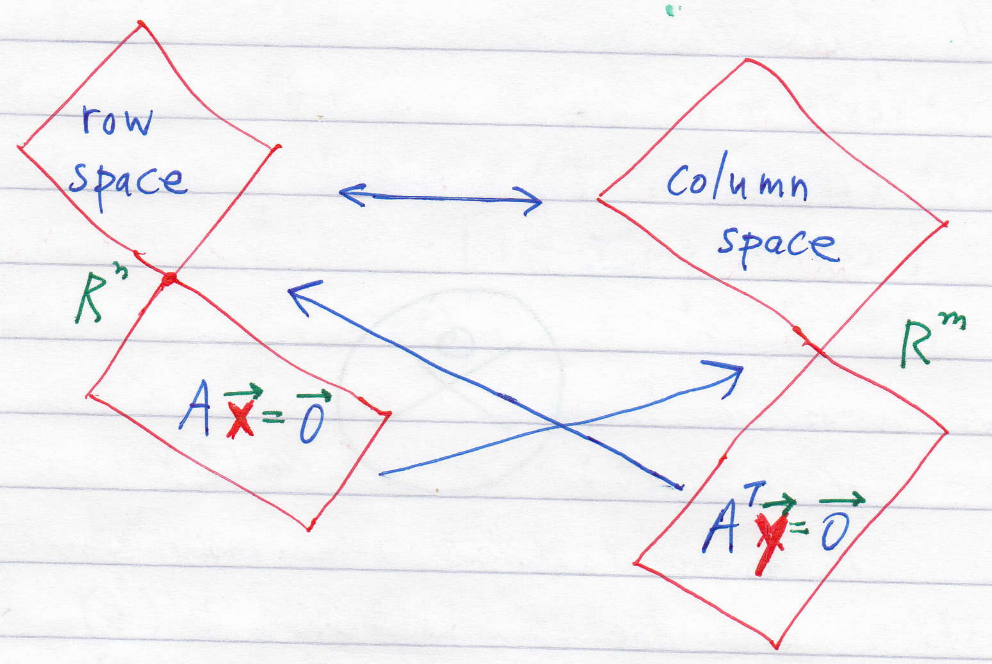
if $x = f(\theta)$ and $y = \phi(\theta)$

it is evident that θ is an inverse function of x.

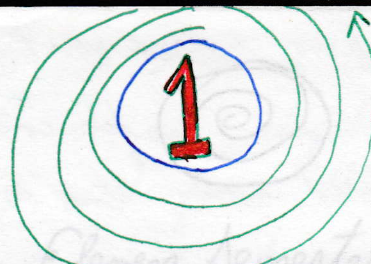
Then
$$\frac{dy}{dx} = \frac{dy}{d\theta}\cdot\frac{d\theta}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\phi'(\theta)}{f'(\theta)}$$

This is the slope at point (x, y) found when the parametric equations are given.

On 13th January, 2000, I decided to begin a separate series of logbooks for technical cogitations called "TECHNICAL LOGBOOKS" which will be a continuation of the "NOTES ON MATHEMATICS" series. BRAINWAVES notes will continue to represent my "Notes From Formal Lectures". TECHNICAL LOGBOOKS will consist of "memos" (not "cogitations")



" new tricks "
for the bearded one



AN OLD DOG LEARNING NEW TRICKS

2000.042.5 →	$YYYY.DDD.D_W$ →	year. julian day. day of week
02.11.0000	MM.DD.HHMM	month. day of month. time of day

Instructors for January to May, 2000 C.E.

Levitt	Multivariable Calculus (Calculus III)	A
Sanderson, Yasmine	Linear Algebra	B+
Tierney, Myles	Mathematical Reasoning	B+

– a wonderfully challenging term at Rutgers, the State University of New Jersey – I have been blessed by the State with tuition {20 credits per year}, books, meals, room {fall, winter, spring – not summer (oh well)}. I have an associates degree in Computer Science. I am working on a bachelors degree in Computer Science, with a minor in Mathematics – AWESOME!

2

JUST GETTING BY, BUT GETTING BY

2000.049.5
02.18.1900

Pulling off the impossible: How does a janitor_{1997} (age 30) become a Computer Scientist / Mathematician? It does not happen overnight. Even with the very beginnings... college prep education from 1981 to 1985 (age 15 to 18), because of the 9 year gap in his education, the janitor_{1993} began studying mathematics in his free time, much to the dismay of his sexy live in girlfriend, Sherry Nevulis$_{\text{his age minus 5 years}}$.

You can't lose brushing up on College Algebra, Trigonometry, and even Analytic Geometry. By 1994, this organism became disgruntled $\text{janitor/student}_{1994-1996}$. From 1995 into 1997, there was a fall, a crisis, an ending, a beginning, a psychotic episode, an arrest, et cetera.

By July 1997 the organism was no longer employed as a janitor, no longer employed at all, and it had stopped attending college after Pascal class in 1996. By 1998, the student was fulltime. By 2000, the $\text{student}_{\text{COMMUNITY COLLEGE}}$ became the $\text{student}_{\text{RUTGERS UNIVERSITY}}$

gnosi, by my definition is a "knower of obscure knowledge"

There are some keys to understanding this word, gnosi, as I understand it.

gno (KNOW)

-sis (a suffix appearing in loanwords from Greek, where it was used to form - from VERBS - "abstract NOUNS" of action)

Key to definition: knower of OBSCURE KNOWledge

obscure: not readily seen
far from public notice
far from worldly affairs
remote
lacking in light, dark
enveloped in, or concealed by, darkness
frequenting darkness
unknown
to conceal or conceal by confusing
secluded, unnoticed

in a sentence: A gnosi knows ~~the~~ the unknown.

note: rabbi (rabī)

Hebrew rabbi - my master
rabb master + -i my.

gno si. Gnosi MWH.

Note about "the beautiful female" in Calculus III.
Yes, she was even more stunning than ever.
She is BEAUTIFUL! But now I am
finding anger to be a great source of
mental energy. I am ANGRY that I
do not have time to really understand the
concepts. I am ANGRY that I have
limits to how much I can learn at one
time. I am ANGRY as a direct cause
of sexual frustration. Seeing (and not being
able to pursue) her causes sexual
frustration = anger.

Also frustrated with LIMITATIONS of
my mental capacity. I am honestly
struggling to understand. I am slipping
behind — panic attack — eye twitching.
I am angry at people who think they
KNOW everything. No longer will I let such
blockheads get to me. (the Noès, Desjardin's, Sandles...)

Some notes from logbook #59 (Scientia est Potentia):

$\frac{d}{dx}(\ln(x)) = \frac{1}{x}$; very well, what about $\int \ln(x)\,dx$?

not as simple as just saying $\frac{1}{x}$!!!

We "mathematicians" use the technique that goes by the name of INTEGRATION BY PARTS.

$\int \ln(x)\,dx$ is worth reviewing.

$\int u\,dv = uv - \int v\,du$

let $u = \ln x \quad dv = dx \quad du = \frac{1}{x}\,dx$;

and $v = \int dv = \int dx = x$

Substituting, we obtain $\int \ln(x)\,dx =$

$(\ln(x))x - \int \frac{x}{x}\,dx = x\ln(x) - x + C$

054.0200 I sent my nephew some email to try to cheer him up and guide him to the tools that he may wish to use to "work on himself" himself (REBT). We spoke for about an hour via Internet. Now, rather than start the preexam (I will start first thing tomorrow), I will relax and skim through logbooks 59 and 60 before I slip into slumber.

Now $\int \frac{1}{x}\,dx = \ln(x) + C \qquad \int x^{-1}\,dx \stackrel{?}{=} \frac{x^0}{0}$ is NOT defined

hence $\ln(x)$ ... but $\int \ln(x)\,dx$ is altogether different.

It is in the form $\int u\,dv$, where $dv = dx$ and $\int dv = v = \int dx =$

Transfer courses		equivalents	Requirement	Pts
1989	Literature of Occult		Human 1 of 4	3
1994	Calculus I	640:135	CS degree	4
1995	Calculus II	640:136	CS degree	4
1995	Digital Programming	198:111 ???	[1] $\boxed{\text{CS degree}}$ 12	4
1996	Pascal		NatSci 1 of 4	4
1998	Speech	?	?	3
1998	Philosophy	730:103	Human 2 of 4	3
1998	Writing	355:101	English 1 of 2	3
1998	PHYSICS I	750:203, 205	CS degree	4
1999	Data Structures	198:112	[2] $\boxed{\text{CS degree}}$ 12	3
1999	Anthropology	070:101	SocSci 1 of 4	3
1999	Psychology	830:101	SocSci 2 of 4	3
1999	C++		NatSci 2 of 4	4
1999	Discrete Math	198:205	[3] $\boxed{\text{CS degree}}$ 12	4
1999	Physics II	750:204, 206	CS degree	4
				53

53 estimated to be transfered

30 not taken (possible to take 4+4+3? I doubt it)

ENV. Science
Computer Fundamentals
Unix
Assembler Language
C

Web Design
Operating Systems Technology
Internet Programming With Java

The Multivariable Calculus text, ch 12, is Partial Derivatives. The first sections sounds very cool: FUNCTIONS OF SEVERAL VARIABLES.

A function, in general, - say, a function f of two variables, is a RULE that ASSIGNS to each ordered pair (x, y) in D a unique real number denoted by $f(x,y)$. The set D is the domain of f and its range is the set of values that f takes on, that is, $\{f(x,y) \mid (x,y) \in D\}$

The graph of f is the set S

$$S = \{(x,y,z) \in \mathbb{R}^3 \mid z = f(x,y), (x,y) \in D\}$$

The graph of $f(x) = y$ is a curve.

The graph of $f(x,y) = z$ is a SURFACE.

$$(x, y, f(x,y))$$

a contour map on which points of constant elevation are joined to form level curves, where level curves are the curves with equations $f(x,y) = k$, where k is a constant (in the range of f)

Sketch the level curves of $f(x,y) = 6 - 3x - 2y$ for $k = -6, 0, 6, 12$

The level curves are $6 - 3x - 2y = k$

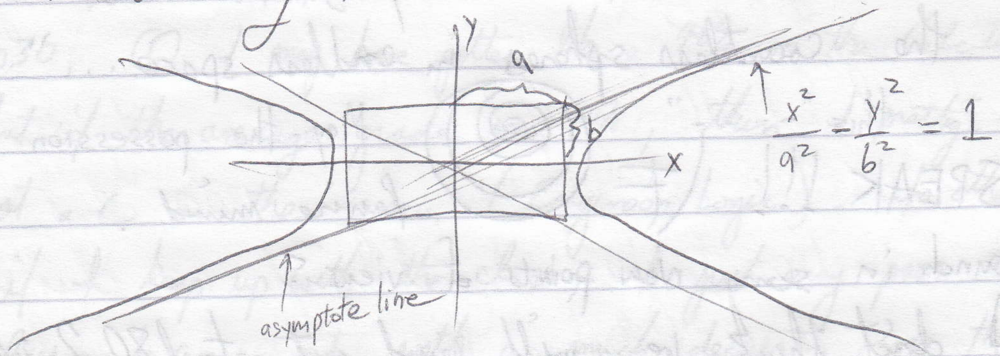
or $3x + 2y + (k-6) = \emptyset$ slope: $-3/2$

① $3x+2y-12=0$, ② $3x+2y-6=0$, ③ $3x+2y=\emptyset$, ④ $3x+2y+6=0$

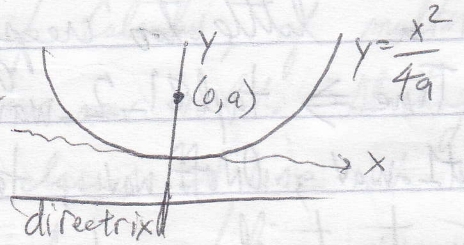
note: hyperbola – the set of all points in a plane such that the difference between the distances to two fixed points is a constant. A hyperbola has two branches that are mirror images of each other. Each branch looks like a misshaped parabola*. The general equation for a hyperbola, with center at origin is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

The meaning of a and b is shown below. The two diagonal lines are called asymptotes. The farther you are from the origin, the closer each part of the curve approaches its respective asymptote line. However, the curve never actually touches the line.



* parabola –



My nephew is distracting me. I will end this session.

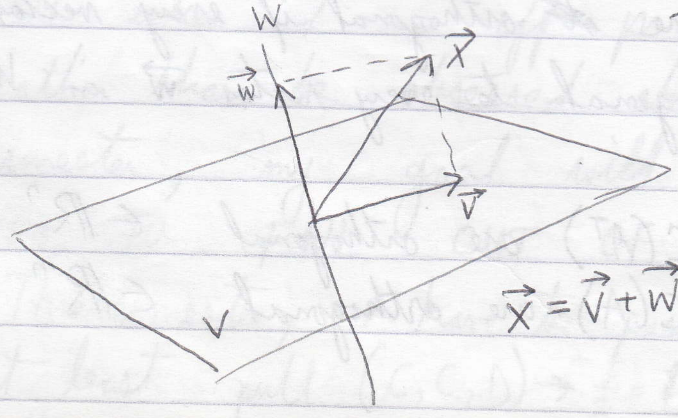
$$N(A) = (C(A^T))^{\perp}$$
$$N(A^T) = (C(A))^{\perp}$$

$$C(A^T) = N(A)^{\perp}$$
$$C(A) = N(A^T)^{\perp}$$

"Lost in Subspaces"
3/2/2000

the line spanned by $(1,0,0)$ is orthogonal to the line spanned by $(0,0,1)$, but V does not equal $W^{\perp}$.
The orthogonal complement of W is a 2-dim subspace containing all vectors of the form $(x_1, x_2, 0)$.
The line $(1,0,0)$ can only be a part of $W^{\perp}$ because its dimension is too small.
If V and W are subspaces of R^n, then any one of these conditions forces them to be orthogonal complements of one another:

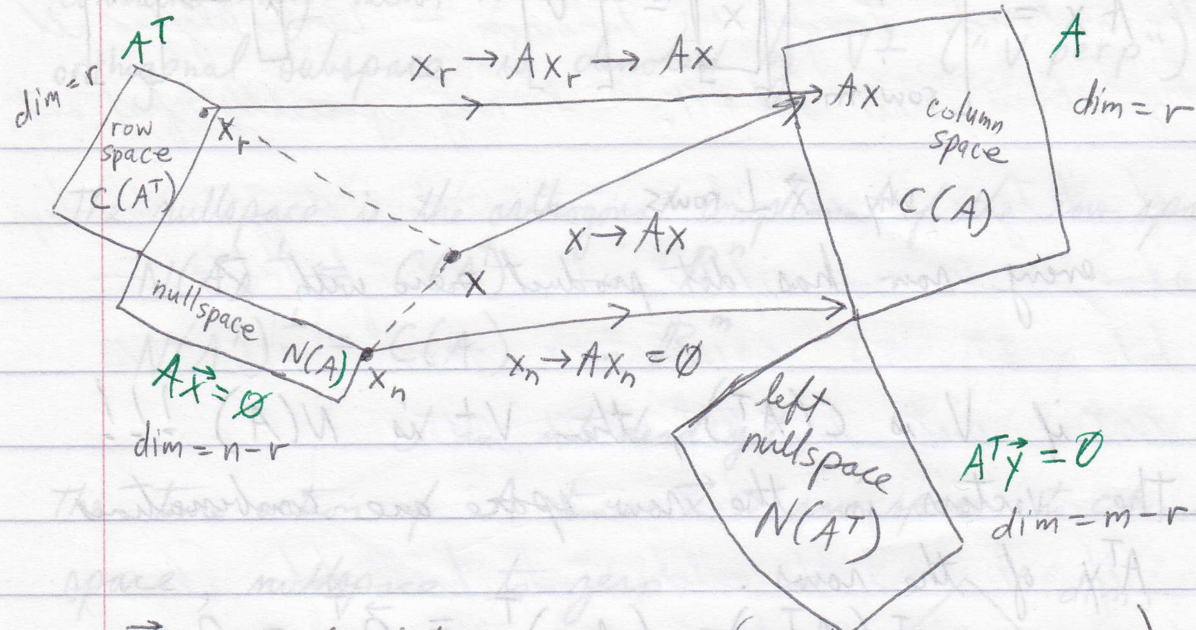
1. $W = V^{\perp}$ (W consists of all vectors orthogonal to V)
2. $V = W^{\perp}$ (V consists of all vectors orthogonal to W)
3. V and W are orthogonal, and $\dim V + \dim W = n$



These components, projection of $\vec{x}$ onto V and W, are orthogonal!
$v^T w = 0$

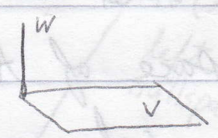
$$\vec{x} = \vec{v} + \vec{w}$$

The true effect of any matrix A :



$\vec{x}$ is split into $x_r + x_n$ (is this x_V and $x_{V^\perp}$?)
and A transforms the row space component $x_r = x_V$ into a vector $Ax_r = Ax_V = Ax$ in the column space, while it transforms the nullspace component $x_n = x_{V^\perp}$ into zero.

floor of room is subspace V
line where 2 walls meet is subspace W
These subspaces are orthogonal
every vector up the meeting line W is $\perp$ to every vector on the floor plane V.
The origin is the corner $(0,0,0)$.



every vector $\vec{x}$ in $N(A)$ is $\perp$ to every row of A.
$N(A)$ and $C(A^T)$ are orthogonal subspaces

$$A\vec{x} = \begin{bmatrix} \boxed{\text{row } 1} \\ \vdots \\ \text{row } m \end{bmatrix} \begin{bmatrix} \boxed{\vec{x}} \end{bmatrix} = \begin{bmatrix} \boxed{0} \\ \vec{0} \\ \end{bmatrix} \qquad \boxed{\text{row } 1} \cdot \boxed{\vec{x}} \to \begin{bmatrix} 0 \\ \\ \end{bmatrix}$$

why $\vec{x} \perp$ rows

every row has dot product zero with $\vec{x}$.

if V is $C(A^T)$, then $V^\perp$ is $N(A)$!!!

The vectors in the row space are combinations $A^T y$ of the rows.

$$x^T(A^T y) = (Ax)^T y = \vec{0}^T y = 0$$

(same thing as above?)

$$A\vec{x} = \begin{bmatrix} 1 & 3 & 4 \\ 5 & 2 & 7 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$ ~~gee~~

The other 2 subspaces: every vector $\vec{y}$ in the nullspace of A^T, $N(A^T)$, is perpendicular to every column of A.

$N(A^T)$ and $C(A)$ are orthogonal.

$$\vec{y}^T A = [\text{---}\ y^T\ \text{---}] \begin{bmatrix} c & & c \\ o & & o \\ l & \cdots & l \\ 1 & & n \end{bmatrix} = [0, \ldots, 0]$$

Definition: The orthogonal complement of a subspace V contains every vector that is $\perp$ to V. This orthogonal subspace is denoted by $V^\perp$ ("V perp")

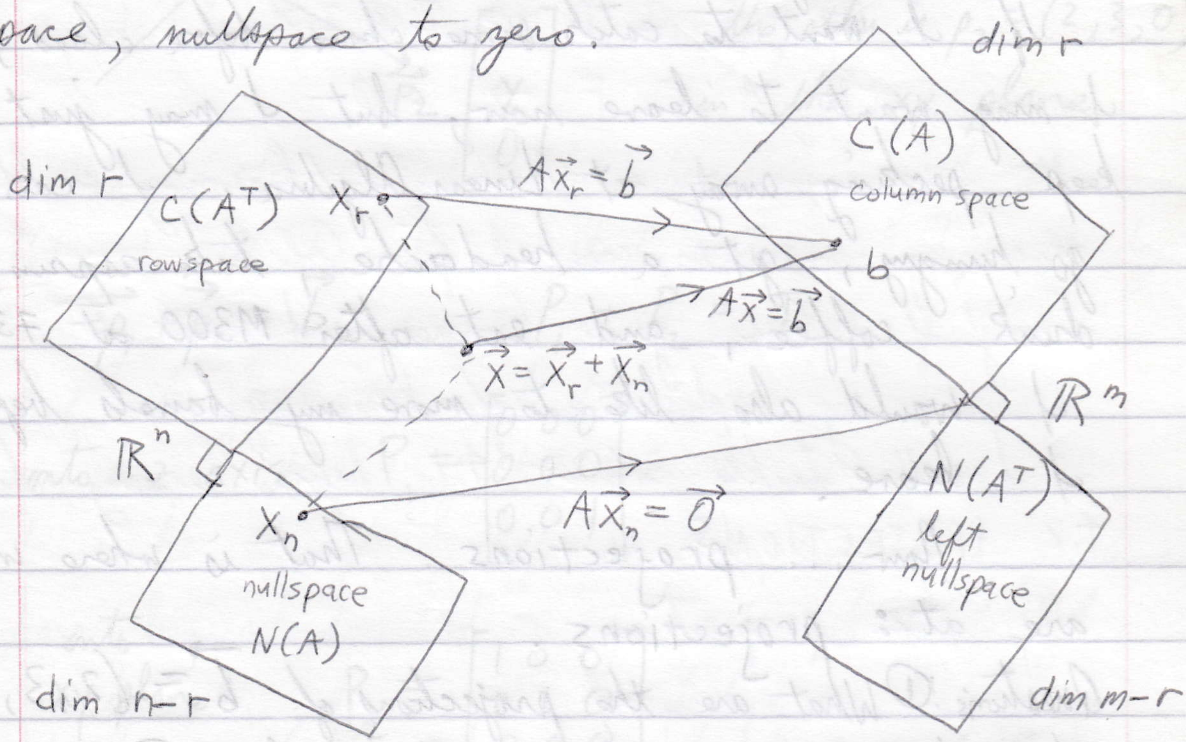
The nullspace is the orthogonal complement of the row space.

$$N(A)^\perp = C(A^T) \text{ in } \mathbb{R}^n$$

$$N(A^T)^\perp = C(A) \text{ in } \mathbb{R}^m$$

I am not dead yet.

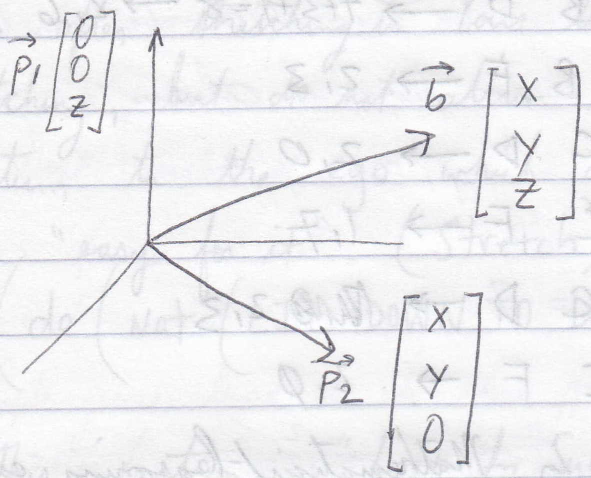
The true action of A times $\vec{x}$: row space to column space, nullspace to zero.



062. 15:50 Before going over for chow at 17:00 or so, I will just finish going through 4.2. Tonight I will try to get through all of 3.6 and start 4.1. Tomorrow and for the rest of the weekend: 4.1, 4.2 of M250; 12.3 to 12.6 M251 ... I will try to climb out of this hole.

There is a matrix P that multiplies $\vec{b}$ to give p. The projection p is Pb.

One projection goes across to the ~~z~~ z-axis. The second projection drops straight down to the xy plane.



start with $\vec{b} = (2,3,4)$

one way up is $p_1 = (0,0,4)$ along the z axis

the other is $p_2 = (2,3,0)$ in the xy plane

$\vec{p}_1 + \vec{p}_2 = \vec{b}$ $\qquad$ $P_1 + P_2 = I$

onto z axis $P_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

onto xy plane: $P_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

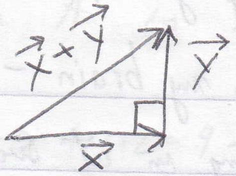
$p_1 = P_1 \vec{b}$ $\qquad$ $p_2 = P_2 \vec{b}$

062.1645 I can't take it anymore. I will get back to this later. I CANNOT SPEND SO MUCH TIME WRITING WHAT IS READ!! STOP!

$\boxed{\begin{matrix} 2000.063.5 \\ 03.03.0000 \end{matrix}}$

unbelievable! Strang goes back to Greek $\longrightarrow$ Pythagoras

Orthogonal vectors



perpendicular vectors: dot product can be written as $x^T y$ (a row times a column, gives the right thing: $x_1 y_1 + x_2 y_2 + \ldots + x_n y_m$)

$\vec{x} \perp \vec{y}$ if $x^T y = \emptyset$

Pythagoras would say ~~" if $\|x\|$ + y^2 ="~~

$\|x\|^2 + \|y\|^2 = \|x+y\|^2 \rightarrow$ orthogonality

"the length squared" $\rightarrow \|\vec{x}\|^2 \rightarrow \vec{x}^T\vec{x}$ is always > 0

It is a length squared $x_1^2 + x_2^2 + \ldots + x_n^2$

$\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $\quad \vec{y} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$ $\quad \vec{x}+\vec{y} = \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$

$\|\vec{x}\|^2 = \left(\sqrt{1^2+2^2+3^2}\right)^2 = 14 \quad \|\vec{y}\|^2 = 5 \quad \|\vec{x}+\vec{y}\|^2 = 19$

My plan is to put down my homework (I completed 4.1) and take notes from Gilbert Strang himself. He is very clear, not at all "in a rush" like my professor. I will go through two full lectures before leaving for Freehold.

$$\vec{x}^T\vec{x} + \vec{y}^T\vec{y} = (\vec{x}+\vec{y})^T(\vec{x}+\vec{y})$$
$$= x^Tx + y^Ty + x^Ty + y^Tx$$

cancel cancel

$$0 = x^Ty + y^Tx$$

Is there a difference between x^Ty and y^Tx?

no, if $x^Ty = 0$ then $y^Tx = 0$

There would be the same $x_1y_1 + x_2y_2 + \ldots$ etc

if $x^Ty + y^Tx = 0$, then $x^Ty = y^Tx$

and $2x^Ty = \emptyset$

Pythagoras lead us to this proof; and 2 is an insignificant constant here, so $x^Ty = \emptyset$

note: It is really cool how something as complicated as Linear Algebra can come down to this idea of orthogonality, $90°$ angles $\rightarrow x^Ty = \emptyset$. !

note: the zero vector, $\vec{\emptyset}$, is orthogonal to everybody.

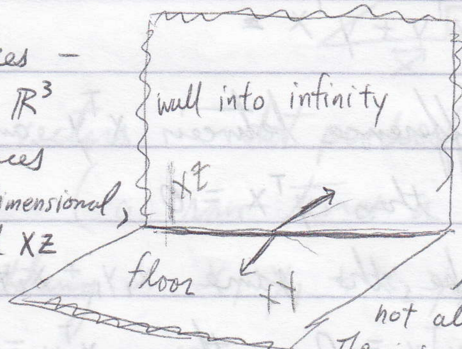
If I can get some STRANG notes down, I can perhaps try to peck away at the beginnings of lab #4 before heading to Freehold for the weekend. I am in awe that my little "prayer" to the universe of man has been answered by Strang himself.

063.0100 Still with Strang lecture 14 . . .

$\boxed{\text{Subspace } S \text{ is orthogonal to subspace } T}$

The difference/connection between orthogonal vectors and orthogonal subspaces –

We are in $\mathbb{R}^3$
The 2 subspaces
are each 2 dimensional,
say, XY and XZ



Are they orthogonal?
What does it mean
for 2 subspaces to
be orthogonal?

not all are 90°
The intersection is in both!

NOT ORTHOGONAL

MEANS

Every vector in S is orthogonal to
every vector in T.

They do not intersect in a nonzero vector, etc.

When is a line through the origin orthogonal to the plane?
Never.

When is a line through the origin orthogonal to $\vec{0}$?
Always.

Neat Fact: row space is orthogonal to the nullspace
"life is great"

These subspaces are "just the right things". They are
cutting the whole space into 2 perpendicular
subspaces. $C(A^T) \perp N(A)$

Why? $A\vec{x} = \vec{0}$ $\quad$ see page 106

$\vec{x}$ is in the nullspace

The equation tells us that $\vec{x}$ is orthogonal to all rows of A.

what else is in the rowspace?

What is that word, "space", telling us?

ALL LINEAR COMBINATIONS OF THE ROWS are also in the rowspace.

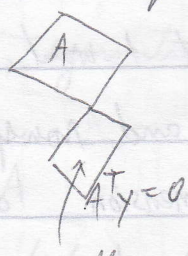
$$c_1 (\text{row } 1)^T \vec{x} = 0$$
$$+ c_2 (\text{row } 2)^T \vec{x} = 0$$

entitled to use scalars and add:

$$\left(c_1 (\text{row } 1)^T + c_2 (\text{row } 2)^T + \ldots \right) \vec{x} = \vec{0}$$

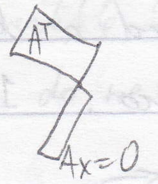
"A^T is just as good a matrix as A"

We have carved up m-dimensional space $\mathbb{R}^m$ into 2 subspaces.



nullspace of A^T

We have carved up n-dimensional space, $\mathbb{R}^n$, into 2 subspaces



note: say we are in $\mathbb{R}^3$, and we have 2 one-dimensional subspaces, two "lines". Could we have a row space that is a line? a nullspace that is a line?

NO. The dimensions are not right.

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 4 & 10 \end{bmatrix} \qquad \dim C(A^T) = 1 = r$$

Geometrically, what does the row space look like?

$$\rightarrow \begin{bmatrix} 1 & 2 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

$n = 3$, $r = 1$, $n - r = \dim N(A) = 2$

$\dim C(A^T) + \dim N(A) =$

$$\begin{bmatrix} 1 & 2 & 5 \\ 2 & 4 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

This can be looked at as $\begin{bmatrix} 1 & 2 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$

$(1 \times 3)(3 \times 1) \rightarrow 1 \times 1$

Gilbert Strang says (this is very funny at 0130): "remember in Calculus there was that dumb normal vector called $\vec{n}$ well, there it is: $(1, 2, 5)$."

"The point I want to emphasize:

" nullspace and rowspace are orthogonal complements

- Their dimension adds to the Whole Space.

you can't have a line and a line as nullspace and rowspace in 3 space: $1 + 1 \neq 3$.

N(A) contains ALL vectors $\perp$ to rowspace

~~The orthogonal complement~~

This has all been sitting there, and now we picked it up. This is Part ~~two~~ of the Fundamental Theorem of Linear Algebra. Part 3 will be about orthogonal bases.

Note: 4.1 is the last chapter on $A\vec{x} = \vec{b}$

We want to "SOLVE" this equation when there is no solution!

What does this mean? Quite simply: $\vec{b}$ is not in the column space.

It is quite typical if A is rectangular that, if m is the number of equations, and n is the number of unknowns, where $m > n$

The rank cannot be m now.

Too many equations $\longrightarrow$ "NOISE" in the right hand side.

There is error in $\vec{b}$, but there is information too!

There is alot of information about $\vec{x}$ in there.

We want to separate the NOISE from the information.

Elimination fails when $0\;0\;0\;0 = \begin{bmatrix} b \\ b \\ \text{not zero} \end{bmatrix}$

$$A^TA$$

What is the nature of this matrix?

$A^T_{n\times m} A_{m\times n}$ is $n \times n$ (square), symmetric: $A^T = A$

$$(A^TA)^T = A^T A^{T^T} = A^TA$$

Is it invertible? If not, what is $N(A^TA)$?

What equation do we solve when we can't solve $A\vec{x} = \vec{b}$

$$\left.A^TA\vec{x} = A^T\vec{b}\;\right\}\text{ central equation of chapter 4}$$

$\vec{x}$ (in $A\vec{x} = \vec{b}$) was the solution if it existed

we will call the $\vec{x}$ in $A^TA\vec{x} = A^T\vec{b}$: $\hat{x}$.

solving $A^TA\hat{x} = A^T\vec{b}$ gives the "best" solution.

063.0200 When is A^TA invertible?

Let's see what A^TA "looks like":

$$\cancel{A\,A} \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$m = 3$, $n = 2$

rank$(A) = 2$, the columns are independent.

"Can we solve it? No way. Linear Algebra is great, but solving 3 equations ($m = 3$ rows) in 2 unknowns ($n = 2$ cols) is impossible.

Only solvable if b is a combination of columns ($\vec{b}$ in $C(A)$), but it usually won't be.

$$A^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 5 \end{bmatrix}$$

$$A^TA = \overset{A^T}{\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 5 \end{bmatrix}} \overset{A}{\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 5 \end{bmatrix}} = \begin{bmatrix} 3 & 8 \\ 8 & 30 \end{bmatrix}$$

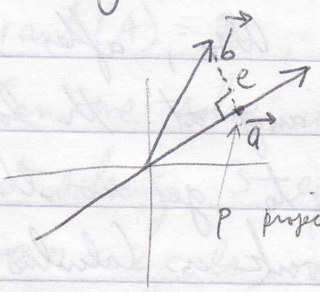
$(2\times3)(3\times2) \to 2\times2$

Plan for Friday 03.03 → take notes herein on lecture 15 (4.2), and – if possible – begin lab #4.

I will have plenty of pencil/paper work and reading of text for the weekend AWAY from computer.

063.0230 Yes, long sleep, but now I am ready. I will not leave for Freehold until the very last minute, say, around 6PM. This gives me about 5 solid hours to use my computer. Lecture 15, start MATLAB. Perhaps Sunday night I will watch lecture 16, which will be about Least Squares approximations.

I may even come back early Sunday.



where does orthogonality come into the picture?

e is the distance from $\vec{b}$ to $\vec{a}$

$e = b - p$

We know that p is some multiple of $\vec{a}$; it is in that 1 dimension subspace of $\vec{a}$: $p = x\vec{a}$

$\vec{a}$ is $\perp$ to e $\longrightarrow$ $\vec{a}^T(\vec{b} - x\vec{a}) = \emptyset$

$\vec{a}^T e = \vec{a}^T(b-p) = \vec{a}^T(b - x\vec{a}) = \emptyset$

$\vec{a}^T b - x\vec{a}^T\vec{a} = 0 \Rightarrow x\vec{a}^T\vec{a} = \vec{a}^T\vec{b}$

$$x = \frac{\vec{a}^T\vec{b}}{\vec{a}^T\vec{a}} \qquad p = \vec{a}x = \vec{a}\,\frac{\vec{a}^T\vec{b}}{\vec{a}^T\vec{a}}$$

Suppose $\vec{b}$ is doubled; then p is doubled.
What if $\vec{a}$ is doubled? What changes? Nothing! The projection is still in the same place. Check the algebra: if a becomes 4a, the fours cancel.

The projection: there is a MATRIX here!
The "projection" is being carried out by some matrix, which we will call "the projection matrix"

The "projection" is some matrix that acts on this $\vec{b}$ and produces the projection $p = Pb$

the projection matrix $\boxed{P = \dfrac{\vec{a}\vec{a}^T}{\vec{a}^T\vec{a}}}$ MATRIX { This is not 1. $\vec{a}^T\vec{a}$ is just a number. $\vec{a}\vec{a}^T$ is a column times a row, $\vec{a}\vec{a}^T$ is therefore a full scale matrix

P $\rightarrow$ $\dfrac{\vec{a}\vec{a}^T}{\vec{a}^T\vec{a}}$ is an interesting matrix. It is the one, that, if multiplied times $\vec{b}$, gives the projection $p = \vec{a}\hat{x}$

Hence $p = \left(\vec{a}\dfrac{\vec{a}^T\vec{b}}{\vec{a}^T\vec{a}}\right) = P\vec{b}$

The rank of this matrix, $\dfrac{\vec{a}\vec{a}^T}{\vec{a}^T\vec{a}} = P$, is ? 1, a rank 1 matrix

The column space $C(P) =$ line through $\vec{a}$

What is the column space? When you multiply any vector $\vec{b}$ by the matrix, you always land in the column space.

A column times a row gives rank 1 matrix. The column is basis for $C(P)$.

Note about "LINEARLY INDEPENDENT":

A set of vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ is linearly independent if it is impossible to find 3 scalars m, n, and p (not all zero) such that $m\vec{a} + n\vec{b} + p\vec{c} = \vec{0}$.

Two vectors are clearly not linearly independent if they are multiples of each other; for example, if $\vec{a} = (2,3,4)$ and $\vec{b} = (20,30,40)$, then $10\vec{a} - \vec{b} = \vec{0}$.

If the vectors are arranged as the columns of a square matrix, then they are linearly independent iff the determinant of that matrix is not zero.

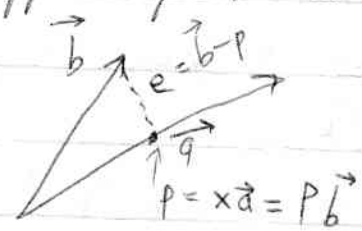
DIMENSION: The dimension of a space is the number of coordinates needed to identify a location in that space.

063.1600 Two more comments about the projection matrix P.

Is P symmetric? $(\vec{a}\vec{a}^T)^T = \vec{a}^{T^T}\vec{a}^T = \vec{a}\vec{a}^T$; $\therefore$ yes.

$P^T = P$. This is a key property.

What happens if I do the projection twice?



the projection twice gives the same projection

$P^2 = P$

It's square is itself.

RECAP:

formula for $x = \dfrac{\vec{a}^T\vec{b}}{\vec{a}^T\vec{a}}$

formula for $p = \vec{a}x = \vec{a}\,\dfrac{\vec{a}^T\vec{b}}{\vec{a}^T\vec{a}} = \left(\dfrac{\vec{a}\vec{a}^T}{\vec{a}^T\vec{a}}\right)\vec{b} = P\vec{b}$

formula for $P = \dfrac{\vec{a}\vec{a}^T}{\vec{a}^T\vec{a}}$

NEXT QUESTION: Why project?

Because $A\vec{x} = \vec{b}$ may have no solutions.

Given a system with more equations than unknowns

Solve the closest problem that we can.

What is the closest? The problem is that $A\vec{x}$ has to be in the column space, and $\vec{b}$ is probably not in the column space

Note: I was able to complete parts 1 and 2 of 4 parts of Lab #4. This is very good. I have until 3/20 to do the rest. I want finish taking notes from Internet: Gilbert Strang.

So we change $\vec{b}$ to what? We choose the closest vector in the column space.

Solve $A\hat{x} = \vec{p}$ instead, where $\vec{p}$ is the projection of $\vec{b}$ onto the column space.

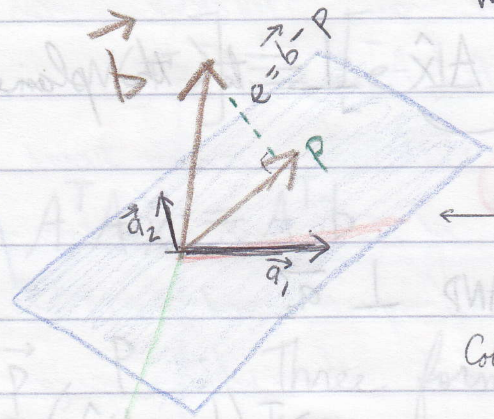
$\hat{x}$ is not the $\vec{x}$ that doesn't exist. $\hat{x}$ is the "x hat" that is the Best Possible Solution.

also "x bar"

$\vec{b}$ is not in the plane.

We project $\vec{b}$ down into the plane

A right angle is crucial.



← The plane of $\vec{a}_1$ and $\vec{a}_2$ $(\vec{a}_1, \vec{a}_2)$. This plane is the Column Space of $A = \begin{bmatrix} \vec{a}_1 & \vec{a}_2 \end{bmatrix}$

What is the plane, a formula for the plane? Given a basis of the plane; 2 vectors a_1 and a_2 form the basis of the plane. They don't have to be perpendicular, but they better be independent.

$(\vec{a}_1, \vec{a}_2)$ "describe" the column space. If $\vec{b}$ were in the column space, the projection would be simply $\vec{b}$.

But most likely there is an error $e = \vec{b} - \vec{p}$

From Geometry and Calculus we know that e is perpendicular to the plane. Our intuition tells us this.

proj p is some multiple of these "basis guys":

$p = \hat{x}_1 \vec{a}_1 + \hat{x}_2 \vec{a} = A\hat{x}$

We have now got a hold of the problem.

The problem is FIND THE RIGHT COMBINATION OF THE COLUMNS SO THAT THE ERROR VECTOR IS PERPENDICULAR TO THE PLANE.

Let us turn these words into an equation:

$p = A\hat{x}$ FIND $\hat{x}$

THE KEY: $\underbrace{b - A\hat{x}}_{e}$ $\perp$ to the plane

$e = b - A\hat{x} \perp \vec{a}_1$ AND $\perp \vec{a}_2$

$$\vec{a}_1^{\,T}(b - A\hat{x}) = \emptyset \quad \text{and} \quad \vec{a}_2^{\,T}(b - A\hat{x}) = \emptyset$$

But we want these in MATRIX FORM.

Look at the matrix $\vec{a}^{\,T}$

$$\begin{bmatrix} \vec{a}_1^{\,T} \text{———} \\ \vec{a}_2^{\,T} \text{———} \end{bmatrix}(b - A\hat{x}) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

This is one way to come up with the equation:

$$A^T(b - A\hat{x}) = \emptyset \implies A^TA\,\hat{x} = A^T\vec{b}$$

Let's get the four subspaces into the picture.

Which subspace is e in? $A^T e = 0$,
therefore e is in the nullspace of A^T:
e in $N(A^T)$. Things that are in the nullspace of A^T are orthogonal to the column space of A.

$e \perp C(A)$ YES!

$$[e \text{ in } N(A^T)] \equiv [e \perp C(A)]$$

$$A^T A \hat{x} = A^T b$$

$\vec{a}^T\vec{a}$ was just a number
• recall $x = \frac{\vec{a}^T\vec{b}}{\vec{a}^T\vec{a}}$

$\hat{x}$, $\vec{p}$, P: Three formulas we are ready for in the n-dimensional case:

$$\hat{x} = (A^TA)^{-1}A^T\vec{b} \; ; \; \vec{p} = A\hat{x} = \underbrace{A(A^TA)^{-1}A^T}_{\text{MAGIC}}\vec{b}$$

In one dimension this "$A(A^TA)^{-1}A^T$" was $\left(\frac{\vec{a}\vec{a}^T}{\vec{a}^T\vec{a}}\right)$; that was projection matrix P.

What MATRIX is multiplying $\vec{b}$ to give the projection?

063.1700 I will just about have enough time to get through this lecture "15 = 4.2". It is just as well I do not go through lecture 16 until Sunday night after I have gone through all excercises and digested the concepts for All Subjects up to this point.

$$P = A(A^TA)^{-1}A^T$$

At first sight, we see $(A^TA)^{-1}$. What would happen if we manipulated $A(A^TA)^{-1}A^T \longrightarrow AA^{-1}(A^T)^{-1}A^T = I$

What is going on here? Somehow, we did something wrong. This was not allowed. A is not a square matrix – It does NOT have an INVERSE, so we must leave $P = A(A^TA)^{-1}A^T$.

A is this matrix with too many rows (equations) and just a couple of columns (unknowns).

If A were nice and square and invertible, then we are projecting $\vec{b}$ onto the whole space – which $\vec{b}$ is already in, so the projection matrix P becomes the identity matrix.

If we are projecting onto a subspace, where A is not square, we cannot perform the inverse, and so it works for both.

Still, $P^T = P$

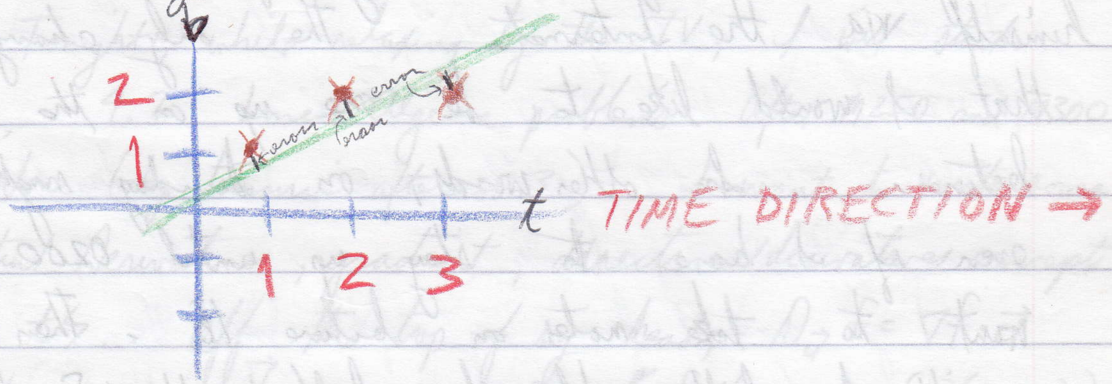
$P^2 = P$

$$(A(A^TA)^{-1}\underline{A^T})(\underline{A}(A^TA)^{-1}A^T) = \cancel{(A^TA)(A^TA)^{-1}}\ \cancel{(A^TA)(A^TA)^{-1}}\ A(A^TA)^{-1}A^T$$

$$P^2 = P \checkmark$$

So, when will there be too many equations and not enough unknowns? LEAST SQUARES FITTING BY A LINE

This will be the material for 4.3 (Lecture 16). I will note it here. Perhaps I may have time to explore section 4.3 in the text book Sunday. Dinner at Mom's with Dad Sunday evening, so I will be in Freehold late Sunday night — and I may not get to Strang's Internet lecture until after my formal lecture with Sanderson 03.06 morning.



Least Squares Fitting By A Line: ~~(b, t)~~ (t, b)

$(1,1), (2,2), (3,2)$

We are looking for this line: $b = C + Dt$

first line	$C + D = 1$	
second line	$C + 2D = 2$	
third line	$C + 3D = 2$	

What is our equation ~~$Ax = b$~~ ?

$$\underbrace{\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}}_{\substack{\text{MATRIX} \\ A}} \underbrace{\begin{bmatrix} C \\ D \end{bmatrix}}_{\substack{\text{UNKNOWN} \\ \vec{x}}} = \underbrace{\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}}_{\substack{\text{RIGHT-HAND SIDE} \\ \vec{b}}}$$

No solution: 3 equations, 2 unknowns.
But we want to find the best solution.
We can solve (by MAGIC): $A^TA\hat{x} = A^T\vec{b}$ GIVES $\left\{ \hat{x},\ p, = P\vec{b},\ P \right.$

$A^T A \hat{x} = A^T \vec{b}$

$$\underset{A^T}{\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}} \underset{A}{\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}} \underset{\hat{x}}{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}} = \underset{A^T}{\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}} \underset{\vec{b}}{\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}}$$

$$\underset{A^TA}{\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}} \underset{\hat{x}}{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}} = \underset{A^T\vec{b}}{\begin{bmatrix} 2 \\ 5 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 3 \end{array}\right] \rightarrow \begin{array}{l} x_1 = -1 \\ x_2 = 3 \end{array}$$

$$\vec{p} = A\vec{x} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} +2 \\ 3 \\ 0 \end{bmatrix}$$

$$\vec{e} = \vec{b} - \vec{p} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} - \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix}$$

This is the same as if I do ~~$A\vec{x}$~~

$\vec{p} = A(A^TA)^{-1}A^T\vec{b}$ - The same answer!

$$\vec{e} = (b - A\hat{x})$$

Tonight I want to get into the section on least squares and approximate solutions, and tomorrow I will get to Relations of M300. Now: Strang lecture 16 Notes

If $\vec{b}$ is in column space, $P\vec{b} = \vec{b}$
if $\vec{b} \perp C(A)$, $P\vec{b} = \emptyset$ } two extreme cases.

$$P = A(A^TA)^{-1}A^T$$

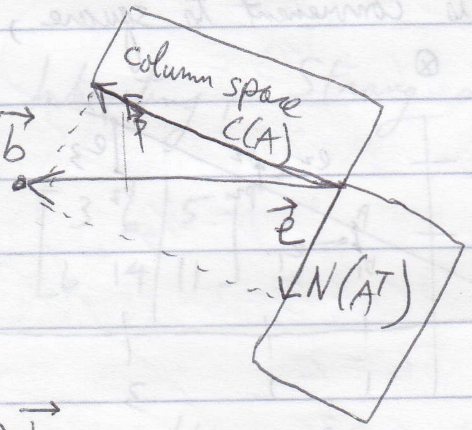
What vectors are perpendicular to the column space?
The vectors in the nullspace of A^T. $C(A) \perp N(A^T)$
if $\perp C(A)$, then in $N(A^T)$ and $A^T\vec{b} = \emptyset$,
hence if $\perp C(A)$, $p = P\vec{b} = A(A^TA)^{-1}\underline{\underline{A^Tb}} = \emptyset$

A typical vector in the column space is a matrix of columns in the form of $A\vec{x}$.
hence if $\vec{b}$ is in $C(A)$, $p = P\vec{b} = A(A^TA)^{-1}A^T\underline{\underline{A\vec{x}}}$

column space

$$p = A(A^TA)^{-1}A^TA\vec{x} = A\vec{x} = b$$

Geometrically, what we are seeing is



$P\vec{b}$

$$\vec{p} + \vec{e} = \vec{b}$$

$$\vec{e} = (I - P)\vec{b}$$

projection onto $\perp$ space

The expression for $\vec{p}$, given a basis for subspace, given matrix A with column vectors are a basis for our column space C(A), $P = A(A^TA)^{-1}A^T$

Recall "find the best straight line" (p. 127).

$(1,1), (2,2), (3,2)$ $\qquad b = C + Dt$

if it went through (1,1): $C + 1D = 1$
if it went through (2,2): $C + 2D = 2$
if it went through (3,2): $C + 3D = 2$

} they do not have a solution, but they have a BEST SOLUTION

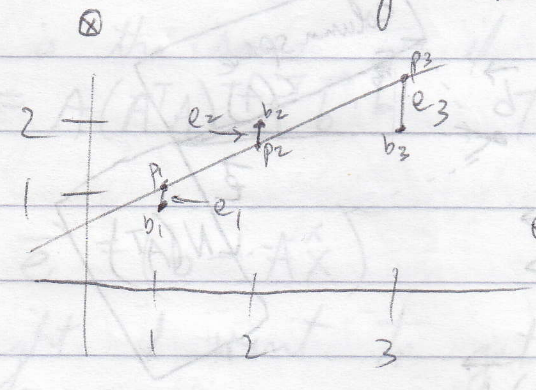
$$\underset{A}{\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}} \underset{\substack{\vec{x} \\ \text{unknown } \vec{x}}}{\begin{bmatrix} C \\ D \end{bmatrix}} = \underset{\vec{b}}{\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}}$$

We want to minimize $(Ax - b) = e$

a sum of squares $\rightarrow$ least squares

minimize the lengths $\|Ax - b\| = \|e\|$

It is convenient to square, hence $\|Ax-b\|^2 = \|e\|^2$



$$\|e\|^2 = e_1^2 + e_2^2 + e_3^2$$

$e = Ax - b$

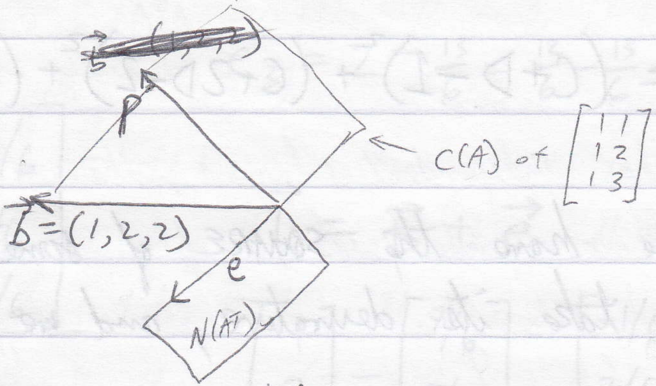
$e_1 = (\underbrace{C + D}_{Ax} - \underbrace{1}_{b})$

$e_2 = (C + 2D - 2)$

$e_3 = (C + 3D - 2)$

Suppose there were a fourth "data point" way up around (0,3) (⊗)
This (0,3) would be called an "outlier" - Ignore it?

What are the points actually on the line? What are p_1, p_2, p_3? If these were the values instead of b_1, b_2, b_3, we would have a solution.



find $\vec{p}$, $\hat{x} = \begin{bmatrix}\hat{C}\\ \hat{D}\end{bmatrix}$

$A^TA\hat{x} = A^T\vec{b}$ } most important equation when dealing with error and NOISE.

$$A^TA = \begin{bmatrix}1&1&1\\1&2&3\end{bmatrix}\begin{bmatrix}1&1\\1&2\\1&3\end{bmatrix} = \begin{bmatrix}3&6\\6&14\end{bmatrix} = A^TA$$

$A^TA = (A^TA)^T$

A^TA is invertible, square, and positive definite

$A^T\vec{b} = \begin{bmatrix}1&1&1\\1&2&3\end{bmatrix}\begin{bmatrix}1\\2\\2\end{bmatrix} = \begin{bmatrix}5\\11\end{bmatrix}$. Interesting, Strang does this:

$$\begin{bmatrix}1&1&1\\1&2&3\end{bmatrix}\left[\begin{array}{cc|c}1&1&1\\1&2&2\\1&3&2\end{array}\right] = \left[\begin{array}{cc|c}3&6&5\\6&14&11\end{array}\right]$$

$\vec{b}$

$3C + 6D = 5$

$6C + 14D = 11$

We could use Calculus as well... see p 138

$A^T[A|b] = [A^TA\ b?]$

Find the minimum by using partial derivatives

$$\|e\|^2 = e_1^2 + e_2^2 + e_3^2$$

$$= (C+D-1)^2 + (C+2D-2)^2 + (C+3D-2)^2$$

If we have the SQUARE of something, and we take its derivative, and we get something linear

$$\frac{\partial e}{\partial C} = 2(C+D-1) + 2(C+2D-2) + 2(C+3D-2)$$

$$= 6C + 2(6D-5) = \boxed{6C + 12D - 10} \Rightarrow \boxed{3C + 6D = 5}$$

$$\frac{\partial e}{\partial D} = 2(C+D-1) + 2(C+2D-2)2 + 2(C+3D-2)3$$

$$= \boxed{12C + 28D - 22} \rightarrow \boxed{6C + 14D = 11}$$

$$3C + 6D = 5$$
$$6C + 14D = 11$$
$$2D = 1$$
$$D = 1/2$$
$$3C + 3 = 5$$
$$3C = 2$$
$$C = 2/3$$

$C = 2/3$
$D = 1/2$

$\therefore$ the best line is

$$\frac{2}{3} + \frac{1}{2}t$$

$$C + Dt$$

$\vec{b} - \vec{p}$

$e_1 = 1 - \frac{7}{6} = \frac{6}{6} - \frac{7}{6} = \frac{-1}{6}$

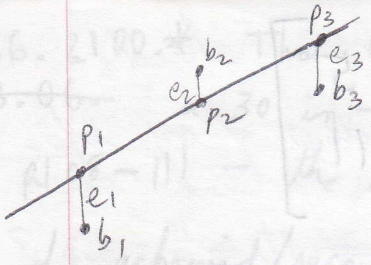
$e_2 = 2 - \frac{5}{3} = \frac{6}{3} - \frac{5}{3} = +\frac{1}{3} = +\frac{2}{6}$

$e_3 =$ [illegible]

$p_1 = \frac{2}{3} + \frac{1}{2}(1) = \frac{7}{6}$

$p_2 = \frac{2}{3} + \frac{1}{2}(2) = \frac{5}{3}$

$p_3 = \frac{2}{3} + \frac{1}{2}(3) = \frac{4}{6} + \frac{9}{6} = \frac{13}{6}$



we want $\vec{p} + \vec{e} = \vec{b} \Rightarrow \vec{e} = \vec{b} - \vec{p}$

$$e_1 = 1 - \frac{7}{6} = -1/6$$

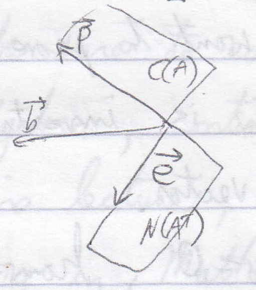
$$e_2 = 2 - \frac{5}{3} = \frac{6}{3} - \frac{5}{3} = \frac{1}{3} = 2/6$$

$$e_3 = 2 - \frac{13}{6} = \frac{12}{6} - \frac{13}{6} = -1/6$$

$$\vec{e} = \begin{bmatrix} -1/6 \\ 2/6 \\ -1/6 \end{bmatrix} \qquad \vec{b} = \vec{p} + \vec{e}$$

$$\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 7/6 \\ 5/3 \\ 13/6 \end{bmatrix} + \begin{bmatrix} -1/6 \\ 2/6 \\ -1/6 \end{bmatrix}$$

$$\vec{p} \perp \vec{e}$$



verify $\begin{bmatrix} 7/6 \\ 10/6 \\ 13/6 \end{bmatrix} \perp \begin{bmatrix} -1/6 \\ 2/6 \\ -1/6 \end{bmatrix}$ $\quad \left(\frac{7}{6}, \frac{10}{6}, \frac{13}{6}\right) \cdot \left(\frac{-1}{6}, \frac{2}{6}, \frac{-1}{6}\right)$

$$= \frac{-7}{36} + \frac{20}{36} - \frac{13}{36} = \emptyset$$

$\vec{e}$ is perpendicular to whatever is in the column space

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \left\} \begin{array}{l} e \perp (1,1,1) = \frac{-1}{6} + \frac{2}{6} + \frac{-1}{6} = 0 \\ e \perp (1,2,3) = \frac{-1}{6} + \frac{4}{6} + \frac{-3}{6} = 0 \end{array} \right.$$

C and D is the combination of the two columns that gives $\vec{p}$. We end up solving the key equation $\left\{ \begin{array}{l} A^T A \hat{x} = A^T \vec{b} \\ \vec{p} = A \hat{x} \end{array} \right.$

lets look at A^TA $\left[\begin{array}{cc|c} \boxed{\begin{matrix} 3 & 6 \\ 6 & 14 \end{matrix}} & & \begin{matrix} 5 \\ 11 \end{matrix} \end{array}\right]$

$\downarrow$

A^TA

is it really invertible ?

If A has independent columns, then A^TA is invertible. It is the independent columns of A that makes the whole process go through.

This is the central fact. We will prove it.

Proof: (by contradiction ?)

Suppose $A^TA\hat{x} = \emptyset$

Now we want to prove that x must be the zero vector.

A matrix is invertible when its nullspace is only the zero vector.

If ~~A^TA is~~ $A^TAx = 0$, how come $x = 0$?

TRICK: Take dot product of both sides with x.

(brilliant idea)

$$x^TA^TAx = \cancel{\emptyset}\, x^T\emptyset$$ "if square is $\emptyset$, then thing is $\emptyset$"

$$x^TA^TAx = \emptyset \Rightarrow (x^TA^T)(Ax) = \emptyset$$

note that $(Ax)^T = x^TA^T \Rightarrow$ so, $(Ax)^T(Ax) = \emptyset$

" if $y^Ty = \emptyset$, then this is length squared."

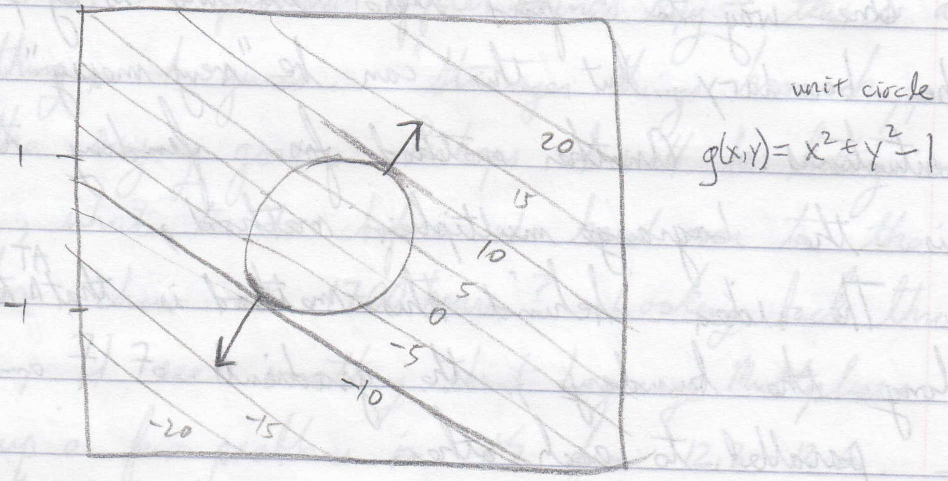
if $(Ax)^T(Ax) = \emptyset$, then $Ax = \emptyset$

If A has independent columns, and $Ax = \emptyset$, then $x = \emptyset$. $\to$ if they are $\perp$ unit vectors

next: Columns are definitely independent if they are perpendicular

067.1700 one last note before I venture over to the feeding pit: level curves are here to stay, so stay focused.

The constraint function here is a unit circle, radius 1, centered at origin. The level curves of $f(x,y) = 6x + 8y$ are the curves defined by $6x + 8y = C$ where C is constant. The value of each C is listed on each level curve. As shown, the function $f(x,y)$ takes on values between -10 and 10 for points on the circle. Hence, max $\to 10$, min $\to -10$.



The maximum and minimum occur at points where the constraint curve is TANGENT to the level curve! These characterizes extreme points: max and min occur at points where normal to constraint curve and normal to level curve point in the same direction! The gradient vector $\vec{\nabla} f = \langle f_x(x,y), f_y(x,y) \rangle$ is normal to the level curve of f through (x,y). It turns out that the normal vector to the constraint curve

is the gradient of g: $\vec{\nabla} g = \langle g_x(x,y), g_y(x,y) \rangle$

Let $z = g(x,y)$. The constraint curve $g(x,y) = \emptyset$ is a level curve corresponding to $\emptyset$. Hence, the gradient vector of $g(x,y)$ on the constraint curve is normal to the constraint curve.

At the maximum and minimum points the normal vectors point in the same direction. That means that the normal vectors are multiples of each other: $\vec{\nabla} f = \lambda \vec{\nabla} g$

Here the unknown multiplier λ is called the Lagrange multiplier. For the case of functions with two variables, the last vector equation can be written:

$$f_x(x,y) = \lambda\, g_x(x,y) \quad \text{and} \quad f_y(x,y) = \lambda\, g_y(x,y)$$

For our problem: $\vec{\nabla} f = \langle 6, 8 \rangle$

$$\vec{\nabla} g = \langle 2x, 2y \rangle$$

Hence, the above vector equation consists of 2 equations: $6 = 2x\lambda$ and $8 = 2y\lambda$

This is 2 equations with 3 unknowns x, y, lambda.

We need a third equation: the constraint equation $g(x,y) = x^2 + y^2 - 1 = 0$

Solving for x and y in the first two: $x = \frac{3}{\lambda}$, $y = \frac{4}{\lambda}$

substituting into the constraint equation:

$$\left(\frac{3}{\lambda}\right)^2 + \left(\frac{4}{\lambda}\right)^2 - 1 = 0 \Rightarrow \lambda = +5, \lambda = -5$$

If $\lambda = 5$, then $x = \frac{3}{5}$ and $y = \frac{4}{5}$

and $f(x, y) = 10 = 6(\frac{3}{5}) + 8(\frac{4}{5})$ MAX of f

If $\lambda = -5$, then $x = \frac{-3}{5}$ and $y = \frac{-4}{5}$ and
$f(x, y) = -10$. MIN of F.

The Lagrange multiplier technique can be applied to problems in higher dimensions. Consider the problem: find the extreme values of $w = f(x, y, z)$ subject to the constraint $g(x, y, z) = 0$.
In this case we get 4 equations for 4 unknowns x, y, z, and lambda:

$$f_x(x, y, z) = \lambda\, g_x(x, y, z)$$

$$f_y(x, y, z) = \lambda\, g_y(x, y, z)$$

$$f_z(x, y, z) = \lambda\, g_z(x, y, z)$$

and the constraint equation:

$$g(x, y, z) = \emptyset$$

For this problem the constraint is a surface in XYZ space.

17:15 hours → I better run... chow before M300 lecture.

067.2045 One thing I love about mathematics is the notation. Understanding the notation, keeping ones wits, amounts to a highly trained manner of reading.

Besides my lustful desires toward or generated by YW_4, I was generally tripped up by the tilde. I was accustomed to $\neg$ and even ! for NOT, but I quickly accepted ~ for NOT.

Now the instructor used the tilde in the following context:

"We write $x \sim y$ for $(x,y) \in R$"

Other sources use the notation xRy to imply $(x,y) \in R$. So $x \sim y$ it is. Now I can decipher my notes from Professor Tierneys ~~quazi quasi~~ cryptic, ~~and~~ quasi-mysterious, and challenging lecture.

I restate the properties of an equivalence relation:

reflexive: $\forall x \in X \quad x \sim x$

symmetric: if $x \sim y$, then $y \sim x$

transitive: if $x \sim y$ and $y \sim z$, then $x \sim z$

If $x \in X \quad [x] = \{ y \in X \mid x \sim y \}$

(is this not backwards from $x \mid x \sim a \rightarrow [a]$?)

Chew slowly.

The equivalence classes of S under R are clearly disjoint or equal since if $x \in [a] \cap [b]$ for $a \neq b$, then $a \sim x$ & $b \sim x$ remember $(a,x) \in R$ and $(b,x) \in R$.

This agrees with Tierney's defn, if $x \in X$ $[x] = \{ y \in X \mid x \sim y \}$ but here $[a] = \{ x \in \text{SOMESET} \mid a \sim x \}$

and $[b] = \{ x \in \text{ANOTHER} \mid b \sim x \}$, hence $x \sim b$. ?

By transitivity $a \sim b$, so $[a] = [b]$.

The set of equivalence classes is a partition of S.

Equivalence classes satisfy:

(1) each $[x] \neq \emptyset$ because x is always equivalent to x

(2) for $x, y \in X$, either $[x] = [y]$ or $[x] \wedge [y] = \emptyset$

(3) each $x \in X$ is in some equivalence class.

This is gone over in my notes. I will go over them once again (without the scent of the erotic yw_4 gripping my central nervous system). I will go over those lecture notes while I smoke my tobacco.

For now, a little more from the "oracle of modern man" — the internet, namely http://www ——

$\sim$

More to tilde than meets the eye.

Evidently, it is not something Myles Tierney dreamed up after all. It symbolizes the definition of relation: $a \sim b \leftrightarrow$ "a and b are related" $= (a,b) \in R$

2000.068.3
03.08.0100

I read through ch 12 text (M251) up to 12.8 Lagrange multipliers, but not including that chapter. I suppose I will go through my notes tomorrow, as well as some worked out problems, transcribing them in t5; afterwhich I will continue lab #4 at question 3. Note also that tomorrow night I will be reviewing my M250 homework as well as all L_{61} notes on linear algebra in preparation for Thursday's quiz. Once I get through this week, I will have time to REGROUP, do M250 xtra credit assignment, make notes in t5, etc.

Before I pick up Russell's <u>Conquest of Happiness</u>, I will jot down the Second Derivative Test for Multivariable Calculus. To Remember the formula for D, it is helpful to write it as a determinant:

$$D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = f_{xx} f_{yy} - (f_{xy})^2$$

suppose second partial derivatives of f are continuous in a disk with center (a,b), and suppose that $f_x(a,b) = 0$ and $f_y(a,b) = 0$ [that is, (a,b) is a critical point of f]

Let $D = D(a,b) = f_{xx}(a,b) f_{yy}(a,b) - [f_{xy}(a,b)]^2$

(a) if $D > 0$ and $f_{xx}(a,b) > 0$, then $f(a,b)$ is a local minimum.

(b) if $D > 0$ and $f_{xx}(a,b) < 0$, then $f(a,b)$ is a local maximum.

(c) if $D < 0$, then $f(a,b)$ is not a local extremum.

Professor Levitt's lecture mentioned nothing of this, but instead, the maximal directional derivative $= |\vec{\nabla} f|$.

"The gradient of f (at any point) is the vector whose direction equals the direction of steepest ascent whose length equals the slope in the direction of steepest ascent."

Let us disect this:

$$\vec{\nabla} f = \langle f_x(x_0, y_0), f_y(x_0, y_0) \rangle$$

Hence, wouldn't $|\vec{\nabla} f| = \sqrt{f_x^2 + f_y^2}$???

At a local maximum or minimum, $\vec{\nabla} f(x_0, y_0) = 0$.

That is $f_x = f_y = 0$ at (x_0, y_0).

Suppose $\vec{\nabla} f(x_0, y_0) \neq \vec{0}$

Then $\vec{\nabla} f \cdot \dfrac{\vec{\nabla} f}{|\vec{\nabla} f|} = \dfrac{|\vec{\nabla} f|^2}{|\vec{\nabla} f|}$ note $\vec{u} \cdot \vec{u} = |\vec{u}|^2$

$= |\vec{\nabla} f| > \emptyset$ Directional derivative of f in $\vec{\nabla} f$ direction.

so f increases in $\vec{\nabla} f$ direction; (x_0, y_0) NOT max

f decreases in $-\vec{\nabla} f$ direction; (x_0, y_0) NOT min

So, local max and min can only occur where $\vec{\nabla} f = \vec{0}$

I will write up an example of such a problem which may show up on a final exam; but I will write it in t^S. It is almost 2AM. Smoke, write one session in t^S, read Russell, sleep.

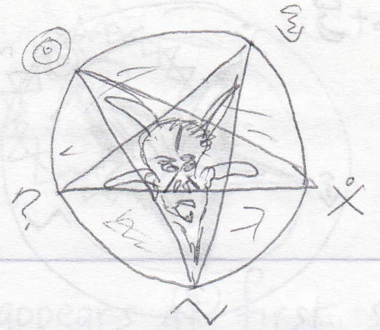
quote here :

"Evil personified appears at first sight repulsive. But the more we study the personality of the Devil, the more fascinating it becomes. The devil is the rebel of the cosmos, the independent in the empire of a tyrant ... he is the individualizing tendency, the craving for originality, which [bodily] upsets the ordinances of God that enforce a definite kind of conduct; he overturns the monotony that would permeate the cosmic spheres if every atom in unconscious righteousness and with pious obedience slavishly followed a generally prescribed course."

— Paul Carus

The History of the Devil and the Idea of Evil

1900

defiance  overcoming fear

$\boxed{\begin{array}{l} 2000.069.4 \\ 03.09.0000 \end{array}}$

What is a rank one matrix? Each row is a multiple of the others. $\text{rank}(\vec{u}\vec{v}^T) = 1$

let $\vec{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ let $\vec{v} = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$

$$\vec{u}\vec{v}^T = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} -1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 2 \\ -2 & 2 & 4 \\ -3 & 3 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} -1 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

next:

$\vec{x}$ is in $N(A)$ iff $\begin{array}{l} \vec{x} \cdot \text{row}_1 = 0 \\ \vec{x} \cdot \text{row}_m = 0 \end{array}$ $\therefore N(A) = C(A^T)^\perp$

$$\vec{y}^T A = \begin{bmatrix} - \vec{y} - \end{bmatrix} \begin{bmatrix} c & & c \\ o & & o \\ l & \cdots & l \\ 1 & & n \end{bmatrix} = \vec{\emptyset}$$

$\vec{y}$ is in $N(A^T)$ iff $\begin{array}{l} \vec{y} \cdot \text{col } 1 = 0 \\ \vec{y} \cdot \text{col } n = 0 \end{array}$ $\therefore N(A^T) = C(A)^\perp$

next: Every vector $\vec{x}$ in $\mathbb{R}^n$ can be written uniquely as $\vec{x} = \vec{x}_r + \vec{x}_n$ where $\vec{x}_r \in C(A^T)$ rowspace and $\vec{x}_n \in N(A)$ nullspace

$$A\vec{x}_n = \emptyset$$
$$A\vec{x}_r = A\vec{x} = \vec{b}$$

example: $A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$ $C(A^T)$:

$z_1\ (1, 0, 1, 0)$

$z_2\ (0, 1, 0, 1)$

orthogonal complement $A^\perp$ is going to be $N(A)$ where the special solutions are $z_3\ (1, 0, -1, 0)$

$z_4\ (0, 1, 0, -1)$.

$\vec{X}$ is in $\mathbb{R}^4$

$$\vec{X} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \vec{X}_r + \vec{X}_n \qquad \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$\underbrace{\frac{a+c}{2}\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + \frac{b+d}{2}\begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}}_{\vec{X}_r} + \underbrace{\frac{a-c}{2}\begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} + \frac{b-d}{2}\begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}}_{\vec{X}_n}$$

This I do not "get". I see where the constant numerators come from, but not the denominator 2.

Just remember that if subspace $V \subseteq \mathbb{R}^n$ and $\dim V = r$, then $\dim V^\perp = n - r$

A basis for $\mathbb{R}^n$ consists of the basis for V together with the basis for $V^\perp$.

This is why we can write $\vec{X} = \vec{X}_V + \vec{X}_{V^\perp}$

069_0100 at this point I can just review my lecture notes, L61 notes, and text book until I sleep.

6

KNOWLEDGE FOR THE SAKE OF KNOWING

At age 33, when one is just starting to work towards a bacholers degree after receiving an associate degree in Computer Science, one is presented with the annoying reality that there exists courses beyond the "undergraduate level". I am speaking of the "Masters" degree, and the other one that makes one a "Doctor" - the PhD or Doctorate.

Academia is high fallootin' and full of snobbery. The poor, yet proud, recipient of the B.S. degree may be made to feel as though he had a Bull Shit degree. I think I will gladly stop at CS BS at age 36. Jesus' Christ - almost 40. Knowledge for the sake of KNOWING! I can't be in this just for some financial reward. I will have some security, but the degree is not a guarentee or fortune. I want to keep this in perspective. I do not want to get tempted by the thought

of training to solve complicated problems with numerical integration on supercomputers... although this still fascinates me, but by the Autumn of 2002, it will have been a full 2 years since Calc 3, and 18 months since Numerical Analysis. I may have a desire to investigate numerical analysis and models of mathematical processes on my own time, after I am settled down with some kind of income as a programmer/analyst/webmaster/technical man; and hence, knowledge will be for the sake of knowing.

As far as the field of Computer Science, I do not regret choosing this as my field— but I must make clear to myself on this 10th day of March, 2000 C.E.: I do not want to be too damn impressed by academia, i.e. the pursuit of massive computing power → solving differential equations with supercomputers, multiple numerical integration to solve problems in physics — fascinating work, but stinks of "Research" and white lab coats. I have to be more practical.

So, I will get some coffee so as to pause; I am overly excited. MANIC PHASE, I guess. I will take some notes from the books I will be investigating over the next couple evenings. By Saturday I will complete the M300 problems, and then I will review my M250 4.3, 4.4 + M251 12.7 work.

tidbits

set $(x_1, x_2, \ldots, x_k)$ of size k.

element x_i of the k-tuple $(x_1, x_2, \ldots, x_k)$ is the ith element (or ith component) of the tuple. Position i is called co-ordinate i. A k-ary relation, R, is a set of k-tuples. When $k = 2$ the relation R is said to be a binary relation, when $k=3$, it is ternary, when $k=4$, quaternary, etc.

revelation about relations: $(<)$, $(=)$, $(>)$, $(\leq)$, $(\geq)$, $(\neq)$ are predefined binary relations. We deal with relations all the time. Ternary relations are needed to define the binary operations $+$, $-$, $*$, $/$, and $\wedge$ $(x \wedge y = x^y)$. The tuple (a, b, c) is a member of the relation $+$ iff $a + b = c$. $\square$

So, "+" is a ternary relation... not so mysterious after all.

cryptic:

$$\underline{\subset} = \{ x \mid x \in \subset \text{ or } x \in = \}$$

note: relations will be exploited in database management as well as discrete structures.

In relational databases, each relation is a set

One may refer to various components of a tuple by the component name, so we speak of the ID-NUM field of a tuple in STAT, the ADDR field of a tuple in BIODATA, etc

BIODATA

NAME	ADDR	PH-NUM	BIRTH	SEX

FIELD

COMPONENT OF tuple

$$(x_1, \quad x_2, \quad x_3, \quad \ldots, \quad x_k)$$

to represent as a matrix (relation matrix)

$$M(i,j) = m_{ij} = \begin{cases} 1 \text{ if } (a_i, a_j) \in R \\ 0 \text{ otherwise} \end{cases}$$

075. 2200 CDM 4.2.4 Equivalence Relations ✓

read text CDM on partial orders.

The relations $<$ and $\leq$ are partial ordering relations as is $\leq_2 = \{ (a,b) \mid$ ~~a~~ a divides b , $a,b \in \mathbb{N} \}$

A record can be seen as an ordered set, a tuple, $(v_1, v_2, \ldots, v_r)$. The record $(v_1, v_2, \ldots, v_k)$ corresponds to the relational scheme RS iff $r=k$ and $v_i \in D_i$.

In the context of databases, relations are usually drawn as a table with each row corresponding to a record (a tuple). Each column is labled by the corresponding attribute name (ith component position i).

An instance of a relation scheme is the specific case. Example:

Relational Database Scheme:

STUDENT-INFO = { BIODATA, FEE, DEPT, COURSE, STAT }

where BIODATA = (NAME, ADDR, PH-NUM, BIRTH, SEX, ID)

FEE = (NAME, ID, QTR, $)

etc... Where BIODATA is a relation scheme and STUDENT-INFO is a set of relation schemes.

CDM STOP. RETURN TO IT when we explore functions in M300.

NEXT: TAM ch 3. READ. revelation. digraphs.

There is an edge from vertex x to vertex y exactly when $(x,y) \in R$. Also if $R = \leq$, then xRy makes much sense. We see $x \leq y$ all the time! LIGHTBULBS ON.

I can construct a digraph for required compsci prereqs

composite of R and S is $S \circ R =$

$\{(a,c) : \exists b \in B \mid (a,b) \in R \text{ and } (b,c) \in S\}$

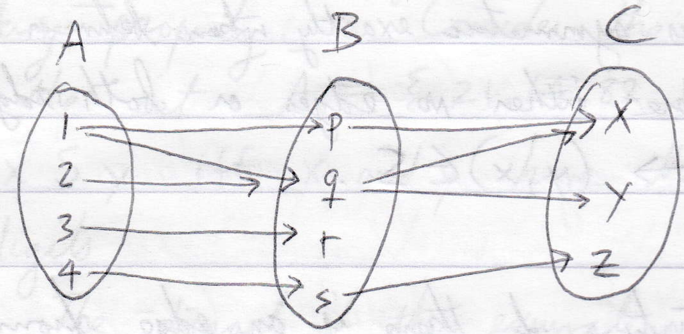
To determine $S \circ R$, remember that the relation R is from the first to the second, and S is from the second to the third.

Let $A = \{1, 2, 3, 4\}$, $B = \{p, q, r, s\}$, $C = \{x, y, z\}$

Let $R = \{(1,p), (1,q), (2,q), (3,r), (4,s)\}$ be a relation from A to B. Let $S = \{(p,x), (q,x), (q,y), (s,y)\}$ be a relation from B to C.

An element a from A is related to an element c of C under $S \circ R$ if there is at least one "intermediate" element b of B.

Example: since $(1,p) \in R$ and $(p,x) \in S$ then $(1,x) \in S \circ R$



$S \circ R = \{(1,x), (1,y), (2,x), (2,y), (4,y)\}$

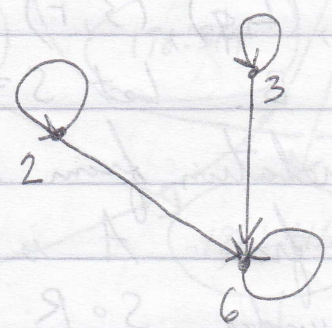
and now, TAM 3.2 Equivalence Relations → after a smoke.

075: 2300 Now we bring in directed graphs — something Myles does not mention, but which may prove beneficial to me.

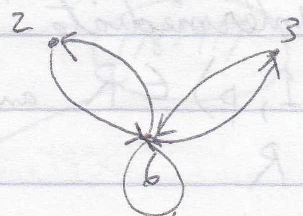
F17 The properties of reflexivity, symmetry, and transitivity can be nicely characterized by properties of the digraph of the relation.

A relation is reflexive exactly when every vertex has a LOOP (an edge from the vertex to itself).

digraph of the "cryptic" $\leq_2$ ("divides") see p. 164



symmetry

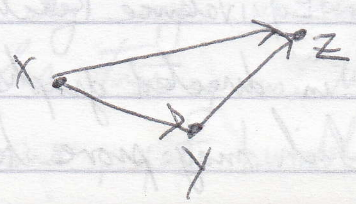


$x+y > 7$

relation S is symmetric exactly when between any two vertices there are either no edges or both edges.

$(x,y) \in S \rightarrow (y,x) \in S$.

For transitivity, if there is an edge from x to y and an edge from y to z, there must be an edge from x to z.



$S = \{x+y > 7\}$ is not transitive because $2 S 6$ and $6 S 3$ but 2 is not S-related to 3.

For $x \in A$, the equivalence class of x determined by R is the set $\{y \in A \mid (x,y) \in R\}$. This is read "the class of x modulo R" or "x mod R".

The set of all equivalence classes is called A mod R.

The relation $H = \{(1,1),(2,2),(3,3),(1,2),(2,1)\}$ is an equivalence relation on the set $A = \{1,2,3\}$

Here 1 modulo H = 2 modulo H = $\{1,2\}$

note: modulo and mod are interchangeable.

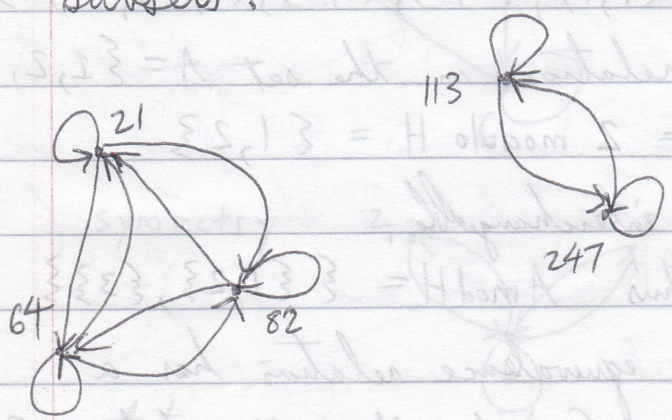
3 mod H = $\{3\}$, Thus A mod H = $\{\{1,2\},\{3\}\}$

The digraph of an equivalence relation has a striking property. Consider the ~~set~~ relation S on the set $A = \{21, 64, 82, 113, 247, 1042\}$ given by x S y iff x and y have the same number of digits.

The digraph of this equivalence relation looks like 3 separate digraphs. A subset of the vertex set of a graph, together with all the edges connecting vertices in the subset, is called a subdigraph. In this case, the three subdigraphs on the vertex sets $\{21, 64, 82\}$, $\{113, 247\}$, $\{1042\}$: each subdigraph

has all possible edges connecting its vertices.
No edge connects a vertex in one of these three subsets with a vertex of another subset. The

digraph below is a union of three components.
The three components are the three equivalence classes, $\{21, 64, 82\}$, $\{113, 247\}$, $\{1042\}$. The equivalence relation gives us a way of partitioning the set into disjoint subsets.



$x \equiv y \pmod{m}$

"x is congruent to y modulo m"

$x \equiv_m y$ iff m divides $(x-y)$

$28 \equiv_5 3$ means $28 \equiv 3 \pmod{5}$

remainder after $\frac{28}{5}$

$4 \equiv 1 \pmod{3}$

3 divides $(4-1)$

5 divides $(28-3)$

m x−y

$10 \equiv 16 \pmod 3$ because 3 divides $10-16$; that is, $10-16 = 3k$

$$x - y = mk \rightarrow x \equiv y \pmod m$$

The equivalence classes of X modulo m is written $[x]$. And now I get into partitions TAM 3.3 after a smoke. I may be prepared to complete M300 problems tomorrow night. I will hold these books for awhile.

The relation $T = \{(5,7), (7,6), (6,6), (5,5), (7,7), (7,5), (6,5), (5,6), (6,7), (4,4)\}$ is an equivalence relation on the set $A = \{4, 5, 6, 7\}$ with equivalence classes $\{4\}$ and $\{5, 6, 7\}$ so the corresponding partition is $A = \{\{4\}, \{5, 6, 7\}\}$

Every partition in turn determines an equivalence relation. Thus, each concept may be used to describe the other.

The method of producing an equivalence relation from a partition is based on the idea that two objects will be said to be equivalent iff they belong to the same member of the partition.

Let $A = \{1, 2, 3, 4, 5\}$ and $\mathcal{B} = \{\{1,2\}, \{3\}, \{4,5\}\}$ be a partition of A. To make an equivalence relation Q on A, we note that since $\{1, 2\} \in \mathcal{B}$, 1 is related to 1 and 2, 2 is related to 1 and 2,

Therefore $Q = \{(1,1), (1,2), (2,1), (2,2), (3,3), (4,4), (4,5), (5,4), (5,5)\}$ is the equivalence relation associated with $\mathcal{B}$.

The method of using a partition to define a relation always produces an equivalence relation, and furthermore, that the set of equivalence classes of the relation is the same as the original partition.

PLAN: return to TAM 4 when we get into functions as relations.

note: It is just after midnight. I will be up at 05:30 to work with my father. Perhaps I will work on the M300 problems tonight even though it is after midnight. This way, tomorrow I can read the other two "research books" at my leisure, taking a minimal amount of notes → reading to deepen my understanding.

Then, on Friday evening, I may bring my M250 and M251 work to Freehold to review, ~~or~~ (if I am working with Dad or not). I will go with the flow.

I think that I am in for a relaxing weekend, as I can Redo problems from M250 4.3, 4.4 and M251 12.7 in Techgnostic Scrullings

$\boxed{\begin{array}{l}2000.077.5\\03.17.\end{array}}$ Professor Myles Tierny suggested we work out our solutions to the M300 problems on scratch papers before writing up our final answers. 231

I will do this "scratch work" directly in my M300 lecture notebook before doing the write ups from problems 14, 15, and 19.

✗

077.0230 That worked well. I finished the M300 problems. This means I finished all my work for all my classes. Now, while in Freehold I can go over my M250 and M251 work in $t\zeta$, and Sunday I can forge ahead...

Now I want to try to extract a formula relating the number of equivalence relations for a set of objects. This will equal the number of possible partitions.

For all cases where $k \geq 2$ there are id_A and $A \times A$.

That is all there are for the case where $k = 2$.

When $k=3$, there are $2 + 3 = 5$

$\{a, b, c\}$ can be split up 3 different ways besides the 2 basics id_A and $A \times A$.

when $k=4$, there are $2+3+4 = 9$!

pattern : $k + k-1 + k-2$

$$f(n) = \sum_{i=2}^{n} k \quad ; \quad f(4) = \sum_{2}^{4} k = 2+3+4$$

for $k \geq 3$

$$f(5) = \sum_{2}^{5} k = 2+3+4+5 = 14$$

I will test $n = 5$. If it is 14, then I will prove it using mathematical induction.

let $A = \{1, 2, 3, 4, 5\}$

1. $\{\{1\}, \{2\}, \{3\}, \{4\}, \{5\}\}$
2. $\{\{1, 2, 3, 4, 5\}\}$
3. $\{\{1\}, \{2, 3, 4, 5\}\}$
4.
5.
6.
7. $\{\{5\}, \{1, 2, 3, 4\}\}$
8. $\{\{1, 2\}, \{3, 4, 5\}\}$
9. $\{\{1, 3\}, \{2, 4, 5\}\}$
10. $\{\{1, 4\}, \{2, 3, 5\}\}$
11. $\{\{1, 5\}, \{2, 3, 4\}\}$
12. $\{\{2, 3\}, \{$
13. $\{\{3, 4\}$
14. $\{\{4, 5\}$
15. $\{\{3, 5\}$
16. $\{\{2, 4\}$

~~17 $\{2, 5\}$~~

~~18 $\{$~~

$\{1\}, \{2\}, \{3, 4\}$

2^n

$n(n-1) - (n-2)$

$n^2 - n - n + 2 =$

$n^2 - 2n + 2 = (n-1)(n-1) + 1$

$n^2 - 2n + 2$

$(n-1)^2 + 1$

$n = 5$: $2 + 5 + 4 + 3 + 2 + 1 = 17 = 4n - 3$

$= 2 + n + n - 1 = 2n + 1$

$n = 4$: $2 + 4 + 3$

$n = 3$: $2 + 3 = 2 + n$

NDPE $2 + \underset{4}{(n-1)} + \underset{3}{(n-2)} + \underset{2}{(n-3)} + n$

$2 + n + (n-1) + (n-2) + (n-3) + 1 =$

$n = 5$: $3 + 4n - 6 = 4n - 3 =$

$n = 4$: $2 + n + n - 1 = 2n +$

1,2,3 4,5,6

1 2 4 3,5,6

1 3 4 2 5 6

1 3 5 2 4 6

1 3 6 2 5 6

1, 2, 6

$6 + 5 + 4 + 3 + 2 +$

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Is there a hint in the power set where a set with n elements $\mathcal{P}(\text{set})$ has 2^n elements?

There is always the null set and the set itself.

These can correspond to the finest and coarsest partitions.

Then there is the singletons in $\mathcal{P}(\text{set})$. These can correspond to when partition has 1 singleton.

What about $\{1\}, \{2\}, \{3,4\}$?

I better correct it.

for $n=4$: $2 + \underset{4}{n} + \underset{3}{(n-1)} + \underset{2}{\cancel{(n-2)}} + \underset{1}{\cancel{(n-3)}} + \underset{3}{(n-1)} + \underset{2}{(n-2)} + \underset{1}{(n-3)}$

$\cancel{= 2 + 7n - 12 = 7n - 10} = 2 + 5n - 7 = 5n + 5$

$2 + \underset{4}{n} + \underset{6}{2(n-1)} + \underset{4}{2(n-2)} + \underset{2}{2(n-3)}$ $= n(n+1) - (n+1)$

$n(n+1) - \cancel{(n+1)} = n^2 + n - n - 1 = n^2 + 1$

does not watch when $n = 3$

$n=5$

$2 + n + (n-1) + (n-1) +$

$\frac{n!}{2} + (n-1) \rightarrow \frac{4!}{2} + 3 = 12 + 3$

$f(n) = \frac{n!}{2} + (n-1) \Longrightarrow f(3) = \frac{6}{2} + 2 = 5$

$f(4) = \frac{4!}{2} + 3 = 12 + 3 = 15$

$f(6) = \frac{720}{2} + 5 = 365$

$f(5) = \frac{5!}{2} + 4 = \frac{120}{2} + 4 = 64$

are the 64 possible partitions of $\{1,2,3,4,5\}$

~~$2 + n + 2(n-1) + (n-2)$~~ My guess is $P(n) = \frac{n!}{2} + (n-1)$

077.0400 I will relax for awhile so as to sleep by 0500. I can sleep until noon and get out by 3PM.